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Transient free convection flow past vertical cylinder with constant heat flux and mass transfer

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Abstract This paper describes a one dimensional unsteady natural convection flow past an infinite vertical cylinder with heat and mass transfer under the effect of constant heat flux at the surface of the cylinder. Closed form solutions of the dimensionless unsteady linear governing boundary layer equations are obtained in terms of Bessel functions and modified Bessel functions by Laplace transform method. The numerical values of velocity, temperature and concentration profiles are obtained for different values of the physical parameters namely, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and time and presented in graphs. Also, skin friction and Sherwood number are shown graphically and discussed. It is observed that the velocity and temperature increase unboundedly with time, while the concentration approaches steady state at larger times.

1. Introduction

Unsteady free convection flow of a viscous incompressible fluid over a heated vertical cylinder has attracted attention of many researchers because of their wide applications in the fields of engineering and geophysics such as nuclear reactor cooling system and underground energy system. In glass and polymer industries, hot filaments, which are considered as a vertical cylinder, are cooled as they pass through the surrounding environment. Goldstein and Briggs [1] analyzed the transient free convection about vertical plates and circular cylinders to a surrounding initially quiescent fluid. They presented analytical solutions for the infinite cylinders by Laplace transform method. Fujii and Uehara [2] compared laminar natural convection along the outer surface of a vertical cylinder with a vertical flat plate on heat transfer. Combined heat and mass transfer on natural convection along a vertical cylinder was studied by Chen and Yuh [3]. Their study covered a wide range of radii and Prandtl numbers. Velusamy and Garg [4] have given a numerical solution for the transient natural convection over a vertical cylinder of various thermal capacities and radii. A numerical solution for transient free convective flow over a vertical cylinder under the combined buoyancy effects of heat and mass transfer was investigated

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Nomenclature

C'	species concentration	R	dimensionless radial co-ordinate
C	dimensionless species concentration	Sc	Schmidt number
D	mass diffusion coefficient	Sh	Sherwood number
Gr	thermal Grashof number	t'	time
Gc	mass Grashof number	t	dimensionless time
g	acceleration due to gravity	T'	temperature
J_0	Bessel function of first kind and order zero	T	dimensionless temperature
J_1	Bessel function of first kind and order one	u	x-component of velocity
J_2	Bessel function of first kind and order two	U	dimensionless velocity along x-axis
k	the thermal conductivity	V	dummy real variable used in integral
K_0	modified Bessel function of second kind and order zero	Y_0	Bessel function of second kind and order zero
		Y_1	Bessel function of second kind and order one
K_1	modified Bessel function of second kind and order one	Y_2	Bessel function of second kind and order two
		α	thermal diffusivity of fluid
Pr	Prandtl number	ν	kinematic viscosity
Q	the rate of heat supplied at the surface	β	volumetric co-efficient of thermal expansion
r	radial co-ordinate measured from the axis of the cylinder	β^*	volumetric co-efficient of expansion with concentration

by Ganesan and Rani [5], while Ganesan and Rani [6] restudied for variable surface temperature. Yücel [7] studied numerically the natural convection over a vertical cylinder in a porous medium, while Hossain et al. [8] presented a numerical solution for flow past a vertical permeable cylinder in a non-Darcy porous medium. Also, Ganesan and Rani [9] studied magnetohydrodynamic effect on flow past a vertical cylinder with heat and mass transfer. Ganesan and Loganathan [10] analyzed heat and mass transfer on the unsteady natural convective flow past a moving vertical cylinder, numerically. They observed that there was a rise in the velocity due to the presence of mass diffusion. Thereafter, Ganesan and Loganathan [11] investigated the combined effects of radiation and chemical reaction by applying numerical method. Rani [12] studied the effects of variable surface temperature and concentration along a vertical cylinder. The boundary layer equations were solved by Crank–Nicolson type of implicit finite-difference method. Mohammed and Salman [13] studied combined convective flow past cylinder. Deka and Paul [14,15] have presented analytical solution of unsteady natural convective flow past an infinite vertical cylinder with constant surface temperature. They solved the governing boundary layer equations by Laplace transform method. Rashidi et al. [16] examined the free convective heat and mass transfer in a steady two-dimensional flow over a permeable vertical stretching sheet in the presence of radiation and buoyancy effects by homotopy analysis method. Machireddy [17] investigated the combined effects of chemical reaction and radiation on flow past cylinder, numerically. Makinde [18], Nandkeolyar et al. [19], Makinde and Tshehla [20] studied the unsteady hydromagnetic flow past vertical flat plate under different physical situations. Makinde [21] studied chemically reacting hydromagnetic unsteady flow of a radiating fluid past a vertical plate with constant heat flux. Again, Makinde [22] investigated the thermal boundary layer of nanofluids over an unsteady stretching sheet with a convective surface boundary condition using Runge–Kutta integration scheme with shooting technique, whereas Rashidi et al. [23] investigated the steady laminar incompressible free convective flow of a nanofluid past a chemically reacting fluid flowing upward facing horizontal plate in a porous medium with heat generation/absorption and the thermal slip boundary condition, wherein optimal homotopy analysis method (OHAM) was employed. Recently, Rashidi et al. [24] studied the buoyancy effect on MHD flow of a nanofluid over a stretching sheet in the presence of thermal radiation by employing shooting technique together with Runge–Kutta integration scheme.

The numerical solutions of convective flow past a vertical cylinder with constant heat and mass flux have been obtained by different authors, for example, Hess and Miller [25], Zariffeh and Daguenet [26], Hossain and Nakayama [27], Ganesan and Rani [28], Ganesan and Loganathan [29–31], Ishak [32] and Chinyoka and Makinde [33]. However, no exact solution on unsteady free convective flow past vertical cylinder with the combined effects of constant heat flux at the surface of the cylinder with mass transfer seems to have been reported and this motivates the present investigation. The unsteady non-dimensional linear governing equations are solved by the Laplace transform technique and employed complex inversion formula to find the inverses of the transformations. Also, attempt has been made to find solutions for larger times to observe for possible occurrence of steady state for different parameters, namely, thermal Grashof number, mass Grashof number and Schmidt number.

2. Mathematical formulation

We consider a unsteady, laminar and incompressible viscous flow past an infinite vertical cylinder of radius r_0. Here, the x-axis is taken along the axis of the cylinder in the vertically upward direction, and the radial co-ordinate r is taken normal to it. Initially, it is assumed that the cylinder and the fluid are at the same temperature T'_∞ and concentration near the surface is C'_∞. At time $t' > 0$, the concentration level near the cylinder is raised to C'_w and heat is supplied at a constant rate