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ORIGINAL ARTICLE

Flow generated by slow steady rotation of a permeable sphere in a micro-polar fluid

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Abstract The analytical study of the flow generated by the slow steady rotation of a permeable sphere in an incompressible micro-polar fluid is considered. Both the flows internal and external to the sphere are coupled. The result will degenerate to independent equations for the case of viscous fluids for the inner and external flows. The flow field in the form of velocity w and micro rotation function φ are obtained in terms of modified Bessel functions and Gegenbauer polynomials. The flow pattern is shown in the form of graphs. It is interesting to note that the velocity and micro-rotation functions within the sphere are constant at distances from the axis of rotation since it represents a rigid body rotation. Effects of physical parameters on the Couple are also shown in the form of graphs.

1. Introduction

The classical problem of steady rotation of a sphere has been attracting many researchers since it is of experimental and theoretical interest and has applications to industry. Mathematical analysis for Stokes flow due to a steady rotation of a sphere in a viscous fluids by Jeffery [1] is the pioneering work in this direction. Since then various similar problems were analyzed by many authors. Kanwal [2] derived an excellent general formula for a couple on an axisymmetric body in terms of a limit on toroidal (angular) velocity for the case of viscous fluids. Dennis et al. [3] have initiated a numerical work on the steady rotation of a sphere in viscous fluid for a wide range of Reynolds numbers. They showed that the torque exerted by the fluid on the sphere is found to be in good agreement with the experimental and theoretical results at low Reynolds numbers. In the middle of 20th century, the behavior of non-Newtonian fluids due to rotation of axisymmetric bodies was studied by many researchers. Thomas and Walter [4] obtained a solution for velocity using regular perturbation technique for rotation of a sphere in a visco-elastic fluid B′ (known as Walters fluid). The Reynolds number is taken as the perturbation parameter in their study. Ramkissoon [5] developed a formula, similar to the one obtained by Kanwal [2], for couple experience by an axisymmetric body in terms of

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Nomenclature

C	cross viscosity parameter
s	couple stress parameter
M	couple stress tensor
$\bar{Q}, \bar{q}$	dimensional (physical) and non-dimensional velocities
r_0	radius of the sphere
R, r	dimensional and non-dimensional distances from origin to a generic point
W, w	swirl, the moment of velocity, in dimensional and non-dimensional form
α, β, γ	material constants known as gyro viscosity coefficients
μ, k, λ_0	material constants known as viscosity coefficients
λ	non-dimensional geometric parameter
θ	angle between z axis and radius vector
$\bar{v}$	microrotation vector
j	micro-gyration
ρ	density of micro-polar fluid
Ω	angular velocity of the sphere
φ	micro-rotation function

swirl function for couple stress fluid. In case of a micropolar fluid (which is similar to couple stress fluid), the analytical study of rotation of a sphere by Lakshmana Rao et al. [6] is worth mentioning and was the first of its kind. Datta and Srivastava [7] have solved the problem of slow rotation of a sphere with fluid source at its center in a viscous fluid. It was found through their analysis that the value of the couple decreases because of the source at the center. Davis [8] derived a formula for force and torque on a rotating sphere close to and within a fluid-filled rotating sphere. Felderhof [9] regarded the Stokes problem of impulsively twisted sphere in an incompressible viscous fluid. Felderhof [10] studied the transient flow caused by a sudden impulse or twist applied to a sphere immersed in a viscous incompressible fluid. Arbaret et al. [11] discussed the effect of shape and orientation on rigid particle rotation and matrix deformation in simple shear flow. A good analysis on axi-symmetric flows due to rotation of arbitrary shaped axi-symmetric body in viscous fluids was presented by Ashmawy [12]. Recently Srivastava [13] discussed the flow due to rotation of axially symmetric body.

The flows past permeable surfaces and flows due to rotating permeable bodies are of great importance in Biological studies and in industrial applications. But the attention paid by the researchers on flows due to rotation of axisymmetric bodies is very little. In this direction, the first Mathematical analysis was by Leonov [14] in the study of flow past a permeable sphere in a viscous fluid. After that Woolfersdrof [15], Padmavathi et al. [16] and Usha [17] also studied the flow past a permeable sphere in viscous fluid. Srinivasacharya and Krishna Prasad [18] attained the flow due to rotation of a spherical container with a porous lining at the center of the sphere. Murthy et al. [19] have discussed the flow generated by the slow steady rotation of a permeable sphere about its axis of symmetry in an incompressible micropolar fluid. Lakshmana Rao et al. [20] studied Stokes Flow of an incompressible Couple Stress Fluid past a porous spheroidal shell. Aparna et al. [21] have studied the flow due to slow rotation of a permeable sphere in a couple stress fluid. The work done by Rashid et al. [22,23], and Sohail et al. [24] for rotating micro-polar fluids under Magnetic field is worth mentioning.

Due to the Mathematical analysis and of practical importance, in the present paper we consider the Stokes flow due to steady rotation of a permeable sphere in an incompressible micro-polar fluid. The velocity and micro-rotation fields are obtained. The couple experienced by the permeable sphere due to the internal and external flow is evaluated. The effect of cross viscosity and geometric parameters on the couple are shown in the form of graphs.

1.1. Statement and Formulation of the problem

The field equations for an incompressible micropolar fluid as proposed by Eringen [13] are

$$div\bar{Q} = 0 \tag{1}$$

$$\rho\frac{d\overline{Q}}{dt} = \rho\bar{f} - \nabla p + k\nabla\times\bar{v} - (\mu+k)\nabla\times\nabla\times\overline{Q} + (\lambda+2\mu+k)\nabla(\nabla\cdot\overline{Q}) \tag{2}$$

$$\rho J\frac{d\bar{v}}{dt} = \rho\bar{l} - 2k\bar{v} - k\nabla\times\bar{Q} - \gamma\nabla\times\nabla\times\bar{v} + (\alpha+\beta+\gamma)\nabla(\nabla\cdot\bar{v}) \tag{3}$$

where $\bar{Q}$ is the velocity vector, $\bar{v}$ is micro rotation vector, p is pressure, ρ is density, J is micro gyration (here taken as constant), λ, μ, k are viscosity coefficients of dimensions $ML^{-1}T^{-1}$ and α, β, γ are gyro viscosity coefficients of dimensions MLT^{-1} and these are subject to the inequalities

$$k \geqslant 0, 2\mu + k \geqslant 0, 3\lambda + 2\mu + k \geqslant 0$$
$$\gamma \geqslant 0,\ -\gamma \leqslant \beta \leqslant \gamma,\ 3\alpha + \beta + \gamma \geqslant 0$$

The stress tensor t_{ij} and the couple stress tensor m_{ij} for micro-polar fluids are given by $t_{ij} = (-p + \lambda div\bar{Q})\delta_{ij} + (2\mu + k)e_{ij} + k\varepsilon_{ijr}(w_r - v_r)$ and $m_{ij} = \alpha(\nabla\cdot\bar{v})\delta_{ij} + \beta v_{i,j} + \gamma v_{j,i}$ where e_{ij} is the rate of deformation tensor and w_k is vorticity vector.

We consider the slow steady rotation of a permeable thin spherical shell of radius r_0 in an incompressible micro-polar fluid, rotating with angular velocity Ω. Let R, θ, φ be a spherical polar coordinate frame with origin at the center of a sphere and Z-axis along the axis of rotation. (h_1, h_2, h_3) are scale factors given by $h_1 = 1$, $h_2 = R$, $h_3 = R\sin\theta$. Since the flow is steady all physical quantities are time independent and the flow is axi-symmetric, and all the quantities are independent of φ. We assume that the body force and body couple terms are absent (see Fig. 1).

We introduce the following non-dimensional scheme

$$\bar{Q} = \bar{q}w_0, R = ra, P = p\rho w_0^2 \text{ and } \bar{v} = \frac{\bar{l}w_0}{\sigma} \tag{4}$$

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