ARTICLE IN PRESS

Alexandria Engineering Journal (2016) **xxx**, xxx–xxx

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Alexandria Engineering Journal

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ORIGINAL ARTICLE

Natural convection flow in a vertical tube inspired by time-periodic heating

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Received 15 May 2016; revised 15 August 2016; accepted 25 August 2016

KEYWORDS

Natural convection;
Fully developed flow;
Time-periodic heating;
Vertical tube

Abstract This paper theoretically analyzes the flow formation and heat transfer characteristics of fully developed natural convection flow in a vertical tube due to time periodic heating of the surface of the tube. The mathematical model responsible for the present physical situation is presented under relevant boundary conditions. The essential features of natural convection flow formation and associated heat transfer characteristics through the vertical tube are clearly highlighted by the variation in the dimensionless velocity, dimensionless temperature, skin-friction, mass flow rate and rate of heat transfer. Moreover, the effect of Prandtl number and Strouhal number on the momentum and thermal transport characteristics is discussed thoroughly. The study reveals that flow formation, rate of heat transfer and mass flow rate are appreciably influenced by Prandtl number and Strouhal number.

1. Introduction

In the course of modeling a real life situation of fluid flow, periodic heat input in a channel has captured the attention of many researchers due to its everyday applications in electrical and electronic appliances such as oven, drying machines, thermostat component, automatic control and microchips. A number of researchers have studied the effect of periodic heat input on time dependent free convection flow in a channel. Chung and Anderson [1] examined unsteady laminar free convection in a channel. Yang et al. [2] studied laminar natural convection with oscillatory surface temperature using the finite difference approach. From miniaturization of electrical and electronic panels, Bar-Cohen and Rohsenow [3] analyzed fully developed natural convection between two periodically heated parallel plates. Wang [4] investigated free convection flow between vertical plates with periodic heat input. He presented the solutions for the temperature and velocity field of the fluid as function of Prandtl number, Strouhal number and indirectly Rayleigh number. He concluded that increasing Strouhal number decreases the unsteady temperature and velocity of the fluid.

In recent past, Jha and Ajibade [5] analyzed free convection flow between vertical porous plates with periodic heat input being an extension of [4] by including suction/injection on the vertical porous plates. They concluded that inclusion of suction/injection has distorted the symmetric nature of the flow considered by [4]. Other research work on periodic heat input includes Jha and Ajibade [6] who investigated the effects of heat generating/absorbing fluid between vertical porous

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.08.025
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Nomenclature

a	radius of the tube	T_1	steady temperature at the tube
A	steady velocity profile	T_2	amplitude of periodic temperature at the tube
B	periodic velocity profile	u	velocity of fluid
F	steady temperature profile		
G	periodic temperature profile	*Greek alphabets*	
g	acceleration due to gravity	α	thermal diffusivity
I_n	modified Bessel's function of first kind of order n. Where $n = 0, 1, 2, 3, \ldots$	β	coefficient of thermal expansion
M	mass flow rate	κ	thermal conductivity
Pr	Prandtl number	ρ	fluid density
r	dimensional radial coordinate	ψ	phase of temperature
R	dimensionless radial coordinate	χ	phase of velocity
St	Strouhal number	τ	skin friction
t	time	ν	kinematic viscosity
T	temperature of the fluid	ω	frequency
T_0	initial temperature		

plates with periodic heat input. Sparrow and Gregg [7] studied nearly Quasi-steady free convection heat transfer in gases. Menold and Yang [8], Nanda and Sharma [9] and Muhuri and Gupta [10] investigated the effect of periodic heating on a single vertical plate with no edge on the boundary layer development. A coupled stress fluid modeling on free convection oscillatory hydromagnetic flow in inclined rotating channel was studied by Ahmed et al. [11] to see the effect of periodic heat input on microchannel. Adesanya [12] studied free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump. He concluded that increase in slip and temperature jump parameters increases the flow velocity and fluid temperature respectively. Also, Adesanya et al. [13] investigated the effect of a transverse magnetic field on the flow of viscous incompressible fluid flowing through a channel subjected to periodic heating using Adomian decomposition method. They found that both skin friction and Nusselt number decrease at the wall with increasing value of magnetic field parameter. Hossain and Floryan [14] studied pressure-driven flow in a horizontal channel exposed to thermal modulations and concluded that the heating results in a significant reduction in the pressure gradient required to drive the flow when compared to a similar flow in an isothermal channel.

In other related works, Shiu and Wu [15] discussed transient heat transfer in annular fins of various shapes with their bases subjected to a heat flux varying as a sinusoidal function of time. Furthermore, Cole and Crittenden [16] investigated Steady-Periodic heating of cylinder using Green's function approach.

The objective of the present investigation was to present a theoretical analysis of natural convection flow in a vertical tube inspired by time-periodic heating of the surface of the tube. Analytical solution of the momentum and energy equations is derived in terms of modified Bessel's function of first kind. Line and contour graphs are plotted to investigate the effect of periodic heating as well as Prandtl number.

2. Mathematical analysis

Consider a natural convection flow of a viscous incompressible fluid in a vertical tube as shown in Fig. 1. Initially, the surface temperature of the tube and fluid is at T_0 and then assumed to be heated periodically to $T(r,t) = T_1 + T_2\cos(\omega t)$. The natural convection flow formation is due to the temperature gradient between the surface of the tube and fluid. Every other parameters are assumed constant except otherwise stated and are presented in nomenclature. The flow is assumed to be fully developed and the viscous dissipation term in the energy equation is also assumed to be negligible.

For small temperature difference, the density of the fluid in the buoyancy term in the momentum equation considered varies with temperature whereas the density appearing elsewhere in these equations is considered constant (Boussinesq's approximation). Under the usual Boussinesq's approximation, the equation of state is assumed to be

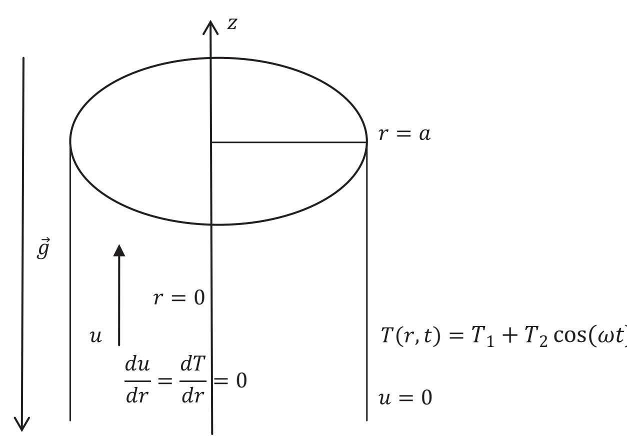

Figure 1 Schematic diagram of the problem.

Please cite this article in press as: B.K. Jha, M.O. Oni, Natural convection flow in a vertical tube inspired by time-periodic heating, Alexandria Eng. J. (2016), http://dx.doi.org/10.1016/j.aej.2016.08.025