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Meshless modeling of bending behavior of bi-directional functionally graded beam structures

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ABSTRACT

Bi-directional Functionally Graded Materials (FGMs) is of more and more focus in special engineering application like aerospace craft at high altitude and speed. In this paper, bending analysis of this type Functionally Graded Beam (FGB) is performed by a proposed meshless Total Lagrangian (TL) Corrective Smoothed Particle Method (CSPM) which owns more capability in accuracy and stability than the classical Smoothed Particle Hydrodynamics (SPH) method. This method is validated by comparison of the bending deformation with analytical method, ABAQUS and a self-programmed finite element code. The influence of the gradient indexes and boundary conditions on the deformation behavior is also investigated. The method is proved to be more precise and stable than the semi-analytical and FE results when dealing with power-law bi-directional FGB.

1. Introduction

Functionally graded materials (FGMs) belong to a kind of advanced composites which are microscopically inhomogeneous and made up of two or more different materials combined in solid states. The volume fraction of each material constituent in FGMs varies gradually with the position along the dimension of the structure, and the properties also change accordingly. Because of the smoothly and continuously varying material properties, FGMs reduce the residual stress and stress concentrations which are fatal defects in traditional laminated composites. Besides, FGMs allow for spatial optimization by grading the volume fractions of two or more constitutes to improve the response of structures. If properly designed, FGMs can bring extraordinary merits such as high resistance to temperature gradients, reduction of thermal stresses, minimization of stress concentration or intensity and high strength to weight ratio. These properties make the FGMs have extensive applications, such as the aerospace, biomedical, energy, optoelectronics, automotive, turbine blade, reactor components [1–3]. With the development of new fabrication technologies, reduction in cost of production, improvement in the properties, FGMs will be applied to more engineering areas.

The advantages of FGMs attract many research interests of scientists and designers, and they have been performed a considerable number of studies during recent years, particularly the mechanical problems in engineering application. There are several ways to understand and predict the mechanical properties of Functionally Graded (FG) structures, including analytic and numerical methods.

A two-dimensional analytical solution based on linear elasticity theory is used to investigate the stress concentration reduction in a non-axisymmetric loaded metal-composite joints where FGM is made as the inter-layer [4]. Based on a two-dimensional theory of elasticity, a governing equation of Functionally Gradient Beam (FGB) under tension and bending is derived by means of the Airy stress function method together with the strain compatibility equation [5]. Sankar [6] investigated the simply supported FGB that subjected to sinusoidal transverse loading, and found that the FGB theory is valid for long slender beams with slowly varying transverse loading. The level of stress concentration occurred in FGB depends on whether the softer or harder face of the beam is loaded. Li et al. [7] used mathematical similarity and load equivalence between the governing equations, and simplified the bending solutions of the FG Timoshenko beams as the calculation of the transition coefficients which can be easily determined by the variation law. The bending response of FG plate resting on elastic foundation and subjected to hygro-thermo-mechanical loading was

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investigated using a four variable refined plate theory without using shear correction factors [8]. Meziane et al. [9] studied the buckling and free vibration of exponentially graded sandwich plates under various boundary conditions by employing an efficient and simple refined theory based on nonlinear variations in the in-plane displacements through the thickness.

However, the solution accuracy using analytical method depends upon the geometrical parameters of the beam and the loading conditions, which promotes the application of numerical method for FGMs with complex geometries, especially Finite Element (FE) method.

Based on a third-order approximation of the axial displacement and constant transverse displacement, Kadoli et al. [10] proposed a FE scheme for the static analysis of beams made of metal-ceramic FGMs. A computationally low cost isogeometric FE model is used to study the fully coupled thermo-mechanical behavior of two-dimensional FGB structures, and it is found that the results obtained from the formulation agree quite well with the reference results [11]. Using an extended FE method, the fatigue crack growth problems in FGM have been simulated in the presence of holes, inclusions, and minor cracks under plastic and plane stress conditions for both edge and center cracks [12]. A FE analysis framework is introduced for the free and forced vibration analyses of FG porous beam type structures, and the proposed approach establish a more unified analysis framework which can investigate simple FG porous beams as well as complex structural systems involving mixture of different materials [13].

Another numerical method is meshless analysis which can avoid mesh-related problems of FE method like mesh-distortion and crack propagation, which promotes its application in FGMs. A meshfree method with a modified distribution function of Moving Kriging interpolation is investigated, and this method is then combined with a high order shear deformation theory for static, dynamic and buckling analyses of FGM isotropic and sandwich plates [14]. Using a meshless Smoothed Hydrodynamic Particle (SPH) method, Lin et al. [15] simulated geometrically nonlinear bending deformation of FGBs with variable thickness, and analyzed the influence of the gradient index and the height ratio of the two edges on the bending behaviors of the FGB. Ching and Yen [16] used the meshless local Petrov-Galerkin method for two-dimensional FG solids which is subjected to either mechanical or thermal loads, and analyzed the bending behavior of the link bar, circular cylinder and simply supported beam. Andrew et al. [17] investigated the two-dimensional steady-state free and forced vibration of FGBs used the element-free Galerkin method and proposed a methodology to optimize the natural frequencies of FG structures by tailoring their material distribution.

It is seen from the above literature survey that the most of the analyses are related to FGMs with material properties which vary in one direction. However, this conventional FGMs are not efficient to fulfill the technical requirements such as the temperature and stress distributions in two or three directions in some practical engineering applications [18–20], for example the high temperature field with extreme temperature gradient on the surface and through the thickness of the fuselage when the aerospace craft flights in high altitude and speed [21]. To eliminate the mentioned drawbacks of the conventional FGMs, a new type of FGMs whose material properties can vary in two directions has attached considerable attentions of researchers and engineers.

Nemat-Alla [22] introduced the concept of adding a third constituent material to the conventional FGM in order to significantly reduce the thermal stresses in machine elements that are subjected to severe thermal loading. They found that this special bi-directional FGM has shown more capability of reducing thermal and residual stresses than one-directional FGM. In Ref. [23], the differential equations of a bi-directional FG Timoshenko beam are established using the Hamilton's principle, the exact dynamic stiffness matrix is formed and the frequency is solved by the Wittrick–William algorithm and a non-iterative algorithm. The isogeometric analysis method for the free vibration analyses of bi-directional FG Timoshenko beams was presented, and through a series of examples it was proven computationally low cost in a variety of bi-directional FGB types [24]. Rad and Alibeigloo [25] investigated the effects of two-parameter linear elastic foundations on axisymmetric response of bi-directionally FG circular plates by using a differential quadrature method. The influences of gradient indices, thickness to radius ratio, foundation stiffness, and edge supports on the static behavior of a plate are studied in the numerical examples. In the investigate of Karamanli [18], the elastostatic behavior of bi-directional FGBs subjected to various sets of boundary conditions are analyzed for the first time by using the Euler-Bernoulli, Timoshenko and Reddy-Bickford beam theories. Using the state space-based differential quadrature method, Lu et al. [26] have presented elasticity solutions for bending and thermal deformations of FGBs with various end conditions, and investigated the influences of material gradient indices on the response of bi-directional FGBs.

As it is seen from the above literature, there are few studies using FEM and no study using meshless method reported for analysis of mechanical response of bi-directional FGBs. Therefore in the present study, as the first time, the elastostatic deflection of bi-directional FGBs is solved by SPH method, which is a true meshless and Lagrangian collocation method without using the fussy mesh connectivity in FEM and background cell in other meshless methods. The effects of the gradation indexes and boundary conditions on the bending behavior of bi-directional FGBs are considered. The solutions are validated by comparing the results with the analytical solutions in previous literature [18], [26] and the FE results by commercial software ABAQUS and self-programmed FE code.

The following content is organized as: the SPH method is introduced and improved in detail in section 2; section 3 presents the homogenization of the bi-directional FGM properties and the governing equations; SPH implementation for solving the deformation of bi-directional FGB is expressed in section 4; in section 5, some numerical examples are dealt with the proposed method.

2. Brief introduction of SPH method

SPH method is a meshless Lagrangian method proposed by Lucy [27], Gingold and Monaghan [28] to solve astrophysical problems in three-dimensional open space at the earliest. Nowadays SPH has been successfully applied to analyze transient fluid [29–31] and solid mechanic [32,33] problems due to its simplicity and ease of applicability. However, it is always subjected to two intrinsic defects, namely "inconsistency" and "tensile instability" problems.

2.1. Insistency problem and corrected smoothed particle method

The insistency problem is inaccuracy of the SPH particle approximation of a function and its derivatives, especially at the particles on the boundary. To ensure the zero-th order consistency, the smoothing function has to satisfy the normalization condition. Since the support domain is truncated by the boundary, this condition cannot be ensured for the particles located on the boundaries or nearby.

An effective method to solve this inconsistency problem is Corrective Smoothed Particle Method (CSPM) proposed by Chen et al. [35,36], which can reproduce the derivatives of any order for a function. It has been proved the capacity to model any unsteady boundary value problem with the Dirichlet and/or von Neumann types of boundary conditions. The following details of CSPM scheme for a function $u(\boldsymbol{x})$ and its derivatives at point $\boldsymbol{x}_i = (x_i, \ y_i)$ (only two-dimensional beam structure is considered in the present study) by combining Taylor series expansion method,