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Non-linear vibrations of sandwich viscoelastic shells

Lahcen Benchouaf [a,*], El Hassan Boutyour [a], El Mostafa Daya [b], Michel Potier-Ferry [b]

[a] *Laboratory of Mechanics, Energetics, Electronics & Telecommunications, Department of Applied Physics, Faculty of Sciences and Technology, Hassan 1st University, PO Box 577, Settat, Morocco*
[b] *Laboratoire d'étude des microstructures et de mécanique des matériaux (LEM3), UMR CNRS 7239, Université de Lorraine, Metz, France*

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This paper deals with the non-linear vibration of sandwich viscoelastic shell structures. Coupling a harmonic balance method with the Galerkin's procedure, one obtains an amplitude equation depending on two complex coefficients. The latter are determined by solving a classical eigenvalue problem and two linear ones. This permits to get the non-linear frequency and the non-linear loss factor as functions of the displacement amplitude. To validate our approach, these relationships are illustrated in the case of a circular sandwich ring.

1. Introduction

In the mechanical structures field, the viscoelastic material is widely used to reduce vibration and noise in many domains (e.g., aerospace industry). Indeed, it can induce an effective damping especially when it is sandwiched between two elastic hard layers. Generally, the damping properties are characterized by two modal parameters that are the frequency and the loss factor. Many investigations have been carried out on the linear dynamic analysis of viscoelastic structures. A major difficulty in their study is that the stiffness matrix is complex and depends non-linearly on the vibration frequency. The solution yield complex modes and complex eigenvalues whose real and imaginary parts are associated respectively with the frequencies and with the loss factors. Several procedures have been developed to determine these quantities. Analytical methods were devoted to simple structures [1–10], and numerical ones using finite element simulations were introduced to design structures with complex geometries and generic boundary conditions [11–22]. The simplest technique is the modal strain energy method used by Ma and He [12], which defines a rather good estimate of the loss factor from a sort of one-mode Galerkin approximation. One notes that from an engineering viewpoint, the most relevant quantity is the loss factor, which is associated with any mode.

In the case of non-linear viscoelastic structures, only a few investigations have been devoted to take into account the non-linear geometrical effects. For instance, these studies concern sandwich viscoelastic structures with simple geometry as beams or plates [23–26]. As it is well known, the non-linear geometrical effects induce some dependence between the frequencies and the loss factors with respect to the amplitude [25,27]. Recently, Boumediene et al. [28] developed a reduction method based on a high-order Newton algorithm and reductions techniques to determine the modal characteristics of

* Corresponding author.
E-mail address: l.benchouaf@uhp.ac.ma (L. Benchouaf).

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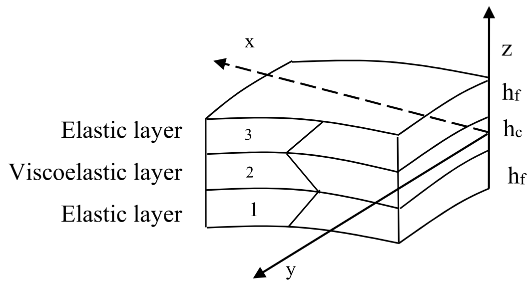

Fig. 1. Geometry of a 3D sandwich structure with two elastic layers and a central viscoelastic one.

viscoelastic sandwich structures. The forced harmonic response of viscoelastic sandwich structures with a reasonable computational cost was also studied, employing a reduction technique and the asymptotic numerical method [29]. Based on von Kármán's theory and taking into account geometric imperfections, the nonlinear vibrations of viscoelastic thin rectangular plates subjected to normal harmonic excitation are investigated by Amabili [30]. Lougou [31] proposed a double-scale asymptotic method for the vibration modeling of large repetitive sandwich structures with a viscoelastic core. In his work [32], Lampoh computes the sensitivity of eigensolutions using a homotopy-based asymptotic numerical method, then a first-order automatic differentiation to study the modeling of the linear free vibration of a sandwich structure including viscoelastic layers yields a complex nonlinear eigenvalue problem. The work of El Khaldi [33] presents a gradient method for viscoelastic behavior identification of damped sandwich structures devoted to the passive control of mechanical vibration.

The aim of this paper is to establish a much simple methodology for the non-linear vibration analysis of viscoelastic shell structures. The approach is based on a coupling of an approximated harmonic balance method with a Galerkin's procedure with one mode. The non-linear modal relationship giving the frequency (free and forced) and the loss factor, with respect to the displacement, are obtained by solving a classical eigenvalue problem and two linear ones [24,27]. To validate our approach, one gives an application to a sandwich viscoelastic ring.

2. Formulation

2.1. Kinematics and constitutive law of the model

Let us consider a thin symmetric sandwich shell having three layers, as shown in Fig. 1; the central layer is viscoelastic and the external ones are elastic. The shear deformation is neglected in the elastic layers, but, it is taken into account in the viscoelastic one; it is induced by the difference between the tangential displacements at the interfaces. For each layer, one denotes by u_i $(i = 1, 2, 3)$ the components of the displacement vector in the z direction and given by:

$$\begin{aligned} &u_i(x, y, z, t) = v_i(x, y, t) + (z - z_i)\beta_i(x, y, t) \quad i = 1, 3 \\ &u_2(x, y, z, t) = v(x, y, t) + z\psi(x, y, t) \end{aligned} \tag{1}$$

where t is the time parameter, (x, y, z) is a coordinate system (z denotes the variation through the thickness). Because of the symmetry, one puts $z_1 = \frac{h_c+h_f}{2} = -z_3$, h_c and h_f being the thicknesses of the central and external layers, respectively. The subscript i indicates the layer variation, starting from the internal layer; 1 and 3 represent the elastic layers, while 2 is associated with the viscoelastic one. β_i and ψ denote the rotations of the cross-section, v_i $(i = 1, 3)$ and v denote tangential components of the displacement vector of the middle planes corresponding to the external and central layers, respectively.

The displacement continuity conditions at the interfaces between the central layer and the external ones permit to get:

$$\begin{aligned} &v_1 = v + \frac{h_c}{2}\psi + \frac{h_f}{2R_1}\beta_1 \\ &v_3 = v - \frac{h_c}{2}\psi - \frac{h_f}{2R_3}\beta_3 \end{aligned} \tag{2}$$

The Green–Lagrange strain in each layer can be decomposed into a linear part and a quadratic one:

$$\gamma_i = \gamma_i(u_i) + \gamma_{nl}(u_i, u_i) \tag{3}$$

For the elastic layers, the behavior is described by the classical Hook law, and it is given, for the viscoelastic one, by the classical convolution product $\otimes$ of the relaxation function $D(t)$ by the time derivative of the deformation:

$$\begin{aligned} &S_i = D(0)\dot{\gamma}_i \quad i = 1, 3 \\ &S_2 = D \otimes \dot{\gamma}_2 \end{aligned} \tag{4}$$

where S_i is the second Piola–Kirchhoff stress tensor corresponding to the layer i and $D(0)$ is the delayed elasticity modulus.

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