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Crack in an elastic thin-film with surface effect

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ABSTRACT

A two-dimensional fracture problem on a crack embedded in an elastic thin-film with surface effect is studied. An elastic analysis of an infinite isotropic homogeneous elastic thin-film with a crack penetrating its thickness is made when subjected in-plane applied loading. Since the elastic thin-film in question is sufficiently thin, the surface stress and surface elasticity are taken into account. First, the principle of virtual work is applied to derive basic equations. Furthermore, coupled governing equations on elastic displacements are obtained, which are then transformed to a single bi-harmonic equation. Mode-I and mode-II cracks are solved and two singular integral equations with Cauchy kernel of the first kind are derived by the Fourier transform technique. Exact elastic field in the whole elastic plane is determined for each case. Fundamental fracture parameters such as the stress intensity factor (SIF) are obtained. Results show that both surface stress and surface elasticity can decrease the SIF, and the SIF is related to material properties. When neglecting surface effects, the SIF is independent of the material properties.

1. Introduction

Ultra-thin structures such as flexible electronics (Eda, Fanchini, & Chhowalla, 2008; Kim et al., 2009), nanomechanical resonators (Choi, Cho, & Kim, 2010b; Dai, Kim, & Eom, 2011), gas separation (Li, Riensche, Menzer, Blum, & Stolten, 2008), water desalination devices (Corry, 2008), etc. have received more and more attention of researchers over the last decade. It mainly relies on the rapid development of advanced micro/nano fabrication techniques. Mechanical properties of such ultra-thin structures are particularly interesting since structural integrity, reliability, and stability are fundamental issues. Some defects such as dislocations, grain boundaries, cracks, holes, etc. inevitably exist in these structures. What is the influence of defects on the mechanical behavior of ultra-thin structures? To date, there are two different approaches to analyze mechanical behaviors. One is based on a point of view according to which an ultra-thin structure may be described by discrete lattices such as atoms, molecules, and even quanta. Conversely the other is based on a point of view according to which an ultra-thin structure may be described by a continuum medium. Recently, Zhang, Li, and Gao (2015) reviewed some of the recent progresses in experimental and theoretical studies on the fracture behaviors of graphene and also formulated several significant issues in this field.

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Along the approach where a thin-film is modelled a continuum with surface effects, many researchers extended the classical elasticity to treat micro/nano-scale materials and structures with crack, where some scale or characteristic parameters are included. An effective approach is to take an increase in the specific surface area of an ultra-thin structure into account, and the surface effects play a crucial role in affecting the mechanical behavior of a beam or plate. For example, Miller and Shenoy reported the classical continuum mechanics to suit for nanosized structural elements by considering surface properties in effective bending stiffness (Miller & Shenoy, 2000). Dingreville, Qu, and Cherkaoui (2005) employed the surface free energy to describe its effect on the elastic behavior of nano materials. Additionally, to study the influence of surface materials on the mechanical behavior of bulk material, Gurtin and Murdoch (GM) (Gurtin & Murdoch, 1975; Gurtin, Weissmuller, & Larche, 1998) introduced surface/interface elasticity along with surface residual stress to extend the classical theory of elasticity. Based on surface elasticity theory, Ru (2010) put forward a simple geometrical explanation of the GM model of surface elasticity and showed several different simplified surface constitutive relations. Duan, Wang, and Karihaloo (2009) gave a review on some progress before 2009 of the classical theory of elasticity being extended to the nanoscale by considering the GM model of surface elasticity. The effects of surface stresses along with surface elasticity on contact problems at nanoscale have been analyzed (Gao, Hao, Fang, & Huang, 2013; Gao, Hao, Huang, & Fang, 2014; Wang & Feng, 2007), and the contact stresses are found to depend strongly on the surface stress and surface elasticity. With the development of nanotechnology, the study of crack problems is particularly significant for better understanding the structural integrity and safety of nano materials and structures. Wu (1999) solved the effect of surface stresses on deformation of an elliptical hole and found the surface stress to change stress intensity factors. Mogilevskaya, Crouch, and Stolarski (2008) employed the complex potential method to address the interaction of elastic fields of multiple circular nano-inhomogeneities or/and nano-pores in a two-dimensional medium using the GM model. Furthermore, Wang, Feng, Wang, and Gao (2008) analyzed the dependence relationship of the crack-tip stresses on surface effects for both mode-I and mode-III cracks and found that when the curvature radius of a blunt crack front decreases to nanometers, surface energy significantly affects the stress intensities near the crack tip. Fu, Wang, and Feng (2008) made a similar analysis for a mode-II nanoscale crack. In addition, based on the complex potential method, Kim et al. examined the effects of surface elasticity in a classical mode-III crack problem for the antiplane shear deformations of a linearly elastic solid or bi-material (Kim, Schiavone, & Ru, 2010; 2011b) and further extended their results to a mode-I and mode-II (interface) crack for plane deformation (Kim, Schiavone, & Ru, 2011a; 2011c). They found that consideration of surface elasticity gives rise to the disappearance of singular stresses near the crack tip. However, a careful examination of the end-point boundary condition showed a logarithmic singularity of stresses and strains near the crack tip when surface elasticity at the crack faces is considered (Kim, Ru, & Schiavone, 2013; Walton, 2012). Wang, Li, Tang, and Shen (2013) employed a double cantilever beam model to deal with the influence of surface stress on stress intensity factors. Nan and Wang (2012) studied the effect of crack face residual surface stress on the fracture of nanoscale materials. In the above-mentioned papers, the surface effects at the crack faces are reflected due to opening of crack, and moreover an infinite long crack or plane strain state is assumed. Nevertheless, most ultra-thin structures in practice such as flexible electronics have sufficiently thin thickness and therefore the surface effects on the upper and lower surface of a thin-film play a key role and should be taken into account. For such a film structure with surface effects, little attention has been paid to surface effects on fracture of a cracked film.

In the present paper, we study an elastic thin-film with a penetrating crack along the film thickness with an emphasis on the effects of surface stress and surface elasticity on fracture parameters. The paper is organized as follows. Basic equations are established according to the principle of virtual work when bulk and surface materials for an elastic film are included in Section 2. In Section 3, the problem considered is formulated and associated boundary value problems are given. Using the Fourier transform technique, the problem is reduced to a singular integral equation and the exact solution of elastic field is derived in Section 4. In Section 5, the influence of surface stress and surface elasticity on fracture parameters such as stress intensity factor are presented graphically. Finally, some conclusions are drawn.

2. Basic equations

Consider an infinite isotropic homogeneous elastic thin film with a penetrating crack. Since the elastic film is sufficiently thin, it can be treated as a plane stress problem. For convenience, it is assumed that an elastic thin-film of thickness h occupies the whole plane region $-\infty < x, y < \infty$. Due to the cause of a sufficiently thin film, the surface effects should be taken into account. Thus we have the following constitutive equations (Lurie & Belyaev, 2005)

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{1}$$

for isotropic bulk material and (Gurtin & Murdoch, 1975)

$$\sigma^s_{\alpha\beta} = \sigma_0\delta_{\alpha\beta} + (\lambda^s + \sigma_0)\varepsilon^s_{\gamma\gamma}\delta_{\alpha\beta} + 2(\mu^s - \sigma_0)\varepsilon^s_{\alpha\beta} + \sigma_0 u^s_{\alpha,\beta}, \tag{2}$$

$$\sigma^s_{\alpha z} = \sigma_0 u^s_{z,\alpha}, \tag{3}$$

for surface material, λ and μ are the Lame constants, λ^s and μ^s the surface Lame constants independent of the surface residual tension, σ_{ij} ($\sigma^s_{\alpha\beta}$) the bulk (surface) stresses, ε_{kl} ($\varepsilon^s_{\delta\gamma}$) the bulk (surface) strains, σ_0 the surface residual stress under unconstrained conditions, u_α elastic displacements. In the above, $\delta_{\alpha\beta}$ is the Kronecker delta function. Latin subscripts *i, j, k*