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Optimal control of an age-structured problem modelling mosquito plasticity

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ABSTRACT

In this paper, we study an age-structured model which has strong biological background about mosquito plasticity. Firstly, we prove the existence of solutions and the comparison principle for a generalized system. Then, we prove the existence of the optimal control for the best harvesting. Finally, we establish necessary optimality conditions.

1. Introduction

Throughout the human history, people have always been combating against many infectious diseases, such as malaria, dengue, yellow and Chikungunya fever, encephalitis and the diseases have caused uncounted mortality of mankind. During the past decades, many researchers studied the pathology of these infectious diseases and tried to control the transmission of them. One of the most studied diseases is malaria, which is mainly transmitted by Anopheles gambiae and Anopheles funestus, the main vectors [1]. As the statistical data show, malaria affects more than 100 tropical countries, placing 3.3 billion people at risk [2] and the life of one African child is taken by malaria every minute [3]. To reduce the suffering of humans from malaria, people have been seeking efficient ways to control the malaria transmission for many years. In the past decades, the control of malaria has made slow but steady progress and the overall mortality rate has dropped by more than 25% since 2000 [4]. The main strategies of controlling malaria are insecticide treated nets (ITNs) and indoor residual spraying (IRS) [1,2,4,5]. However, the effectiveness of these strategies depends on the susceptibility of the vector species to insecticides and their behaviours, ecology and population genetics.

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ITNs and IRS are efficient ways against the main vectors of malaria in Africa. However, the resistance of mosquitoes to insecticides forces them to adapt their behaviours to ensure their survival and reproduction. Especially, they can adapt their biting behaviour from night to daylight [6]. These new behavioural patterns lead to a resurgence of malaria morbidity in several parts of Africa [7]. Thus, new methods are desired to replace the traditional strategies.

In this work, we are going to model mosquito population adaption and study the optimal control problem. We consider a linear model describing the dynamics of a single species population with age dependence and spatial structure as follows

$$\begin{cases} Dp - \delta \Delta p + \mu(a)p = u(a,t,x)p, & (a,t,x) \in Q_{a_\dagger}, \\ p(a,t,0) = p(a,t,24), & (a,t) \in (0,a_\dagger) \times (0,T), \\ \partial_x p(a,t,0) = \partial_x p(a,t,24), & (a,t) \in (0,a_\dagger) \times (0,T), \\ p(0,t,x) = \displaystyle\int_0^{a_\dagger} \beta(a) \int_{x-\eta}^{x+\eta} K(x,s)p(a,t,s)dsda, & (t,x) \in (0,T) \times (0,24), \\ p(a,0,x) = p_0(a,x), & (a,x) \in (0,a_\dagger) \times (0,24), \end{cases} \tag{1}$$

where $Q_{a_\dagger} = (0,a_\dagger) \times (0,T) \times (0,24)$ and

$$Dp(a,t,x) = \lim_{\varepsilon \to 0} \frac{p(a+\varepsilon, t+\varepsilon, x) - p(a,t,x)}{\varepsilon}$$

is the directional derivative of p with respect to direction $(1,1,0)$. For p smooth enough, it is easy to know that

$$Dp = \frac{\partial p}{\partial t} + \frac{\partial p}{\partial a}.$$

Here, $p(a,t,x)$ is the distribution of individuals of age $a \geq 0$ at time $t \geq 0$ and biting at time $x \in [0,24]$, $a_\dagger$ means the life expectancy of an individual and T is a positive constant. As we announced, the mosquitoes can adapt their biting time. Thus, we set their adapting model to be a Δ diffusion with a diffusive coefficient δ. Moreover, $\beta(a)$ and $\mu(a)$ denote the natural fertility-rate and the natural death-rate of individual of age a, respectively. In fact, the new generation is also able to adapt the biting time in order to maximize its fitness. Let η be the maximum biting time difference which the new generation can reach and we model the adaption of the new generation by a kernel K as defined below

$$K(x,s) = \begin{cases} (x-s)^2 e^{-(x-s)^2}, & s \in (0,24), \\ 0, & \text{else}. \end{cases}$$

The control function $u(a,t,x)$ represents the insecticidal effort, such as the use of ITNs and RIs.

In our paper, the main goal is to prove that there exists an optimal control u in limited conditions, that is, u is bounded by two functions ς_1 and ς_2 such that the insecticidal efficiency reaches the best. Since the control function u is negative, it means that we can deal with the following optimal problem

$$(OH) \qquad \text{Maximize} \left\{ -\int_{Q_{a_\dagger}} u(a,t,x)p^u(a,t,x)dtdxda \right\},$$

subject to $u \in U$,

$$U = \{u(a,t,x) \in L^2(Q_{a_\dagger}) |\ \varsigma_1(a,t,x) \leq u(a,t,x) \leq \varsigma_2(a,t,x)\ a.e.\ in\ Q_{a_\dagger}\},$$

where $\varsigma_1, \varsigma_2 \in L^\infty(Q_{a_\dagger}),\ \varsigma_1(a,t,x) \leq \varsigma_2(a,t,x) \leq 0\ a.e.\ in\ Q_{a_\dagger}$ and p^u is the solution of system (1). Here, we say that the control $u^* \in U$ is optimal if

$$\int_{Q_{a_\dagger}} u^*(a,t,x)p^{u^*}(a,t,x)dtdxda \leq \int_{Q_{a_\dagger}} u(a,t,x)p^u(a,t,x)dtdxda,$$