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Stability for a system of two coupled nonlinear oscillators with partial lack of damping

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ABSTRACT

The stability of the null solution of a system of differential equations describing two coupled nonlinear oscillators, one being with lack of damping, is discussed. Under certain assumptions we derive some stability results (see Theorems 3.1 and 4.1), which are in agreement with physical reality.

1. Introduction

In [1] and [2] we reported new stability results for the scalar equation describing a damped nonlinear oscillator and in [3] we discussed the case of some coupled damped nonlinear oscillators. In the present paper, we will study the stability of the null solution to a system of two 1-D coupled nonlinear oscillators, one of which having lack of damping, as shown in Fig. 1.

We will first suppose that both blocks have the same mass, m (for the general case see Remark 3.3), and the spring constants are $k_1(t)$, $k_2(t)$, $k_3(t)$, $t \in \mathbb{R}_+$, where $\mathbb{R}_+ := [0, +\infty)$. We associate with the above physical application the following system of ODEs which describes the motion of the oscillators:

$$\begin{cases} \ddot{x} + \beta(t)\, x - \gamma(t)\, y = 0, \\ \ddot{y} + 2f(t)\, \dot{y} + \delta(t)\, y - \gamma(t)\, x + g(t, x, y) = 0, \end{cases} \quad t \in \mathbb{R}_+. \tag{1.1}$$

The functions $\beta, \delta : \mathbb{R}_+ \to \mathbb{R}$ represent the squares of the phases, depending on the mass and the spring constants $\left(\beta(t) = \frac{1}{m}(k_1(t) + k_2(t)),\ \delta(t) = \frac{1}{m}(k_2(t) + k_3(t)),\ \forall t \in \mathbb{R}_+\right)$ and $\gamma : \mathbb{R}_+ \to \mathbb{R}$, $\gamma(t) = \frac{1}{m} k_2(t)$, $\forall t \in \mathbb{R}_+$ represents the coupling factor of the oscillators. The function $f : \mathbb{R}_+ \to \mathbb{R}$ represents the damping

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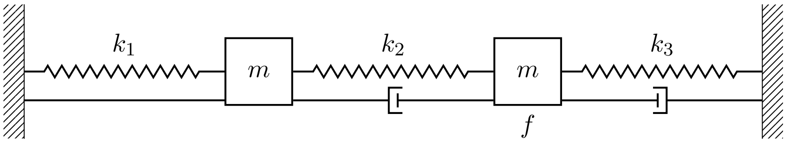

Fig. 1. Two 1-D coupled nonlinear oscillators with partial lack of damping.

coefficient of the second oscillator that is driven by an external force represented by $g : \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and there is no external force acting on the first oscillator.

For extensive studies regarding the stability questions for a single 1-D damped nonlinear oscillator, we refer the reader to [1,2,4–11] and the references therein and for fundamental concepts and results in stability theory we refer the reader to [12–14].

2. General framework

The following hypotheses will be admitted:

(i) $f \in C^1(\mathbb{R}_+)$, $f(t) \geq 0$, $\forall t \in \mathbb{R}_+$, and there exist two constants h, $K \geq 0$ such that

$$\left|f'(t) + f^2(t)\right| \leq K f(t), \forall t \in [h, +\infty);$$

(ii) $\int_0^{+\infty} f(t)\,\mathrm{d}t = +\infty$;

(iii) β, $\delta \in C^1(\mathbb{R}_+)$, β and δ are decreasing,

$$\beta(t) \geq \beta_0 > 0, \delta(t) \geq \delta_0 > 0, \quad \forall t \in \mathbb{R}_+,$$

where β_0 and δ_0 are two constants, and

$$\frac{K}{\sqrt{\delta_0}} < 1;$$

(iv) $\gamma \in C(\mathbb{R}_+)$ and $\gamma(t) \geq 0$, $\forall t \in \mathbb{R}_+$;

(v) $g = g(t, x, y) \in C(\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R})$, g is locally Lipschitzian in x, y, and satisfies the following estimate

$$|g(t, x, y)| \leq f(t)\,\mathrm{o}(|y|), \quad \forall t \in \mathbb{R}_+, \forall x, y \in \mathbb{R},$$

where "o" denotes the usual Landau symbol for $y \to 0$, i.e. $\lim_{y\to 0} \frac{\mathrm{o}(|y|)}{|y|} = 0$.

The following result will be used to prove Theorem 3.1 (see H. Brezis [15, Appendix]; see also [16, Lemma 2.1, p. 47]).

Lemma 2.1. *Let $t_0 \in \mathbb{R}$, $\phi \in L^1(t_0, \infty)$ be a function such that $\phi \geq 0$ a.e. on (t_0, ∞), and p a real constant. If $\omega \in C([t_0, \infty))$ fulfills the inequality*

$$\frac{1}{2}\omega^2(t) \leq \frac{1}{2}p^2 + \int_{t_0}^{t} \phi(s)\,\omega(s)\,\mathrm{d}s, \ \forall t \in [t_0, \infty),$$

then the following inequality holds

$$|\omega(t)| \leq |p| + \int_{t_0}^{t} \phi(s)\,\mathrm{d}s, \ \forall t \in [t_0, \infty).$$