Nonlinear Analysis: Real World Applications 41 (2018) 82–106

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

www.elsevier.com/locate/nonrwa

Bifurcation of limit cycles at infinity in piecewise polynomial systems

Ting Chen[a], Lihong Huang[a,b,*], Pei Yu[c], Wentao Huang[d]

[a] *College of Mathematics and Econometrics, Hunan University, Changsha, 410082, PR China*
[b] *School of Mathematics and Statistics, Changsha University of Science and Technology, Changsha, 410082, PR China*
[c] *Department of Applied Mathematics, Western University, London, Ontario, N6A 5B7, Canada*
[d] *School of Science, Guilin University of Aerospace Technology, Guilin, 541004, PR China*

A R T I C L E I N F O

Article history:
Received 20 April 2017
Received in revised form 2 October 2017
Accepted 7 October 2017

Keywords:
Piecewise polynomial system
Lyapunov constant
Center
Limit cycle

A B S T R A C T

In this paper, we study bifurcation of limit cycles from the equator of piecewise polynomial systems with no singular points at infinity. We develop a method for computing the Lyapunov constants at infinity of piecewise polynomial systems. In particular, we consider cubic piecewise polynomial systems and study limit cycle bifurcations in the neighborhood of the origin and infinity. Moreover, an example is presented to show 11 limit cycles bifurcating from infinity.

1. Introduction

One of the well-known mathematical problems is the second part of Hilbert's 16th problem, which considers the maximal number and relative positions of limit cycles bifurcating in polynomial vector fields of degree n, given by

$$\dot{x} = f_n(x,y), \quad \dot{y} = g_n(x,y), \tag{1}$$

where the dot denotes differentiation with respect to time t. Since Hilbert proposed the problem in 1900, a great deal of works has been done in studying this problem, for example see [1–8]. Let $H(n)$ denote the upper bound of the number of limit cycles that system (1) can have. Chen and Wang [1], and Shi [2] proved the existence of 4 limit cycles with $\{3,1\}$ distribution, i.e., $H(2) \geq 4$. However, this problem is even not

* Corresponding author at: College of Mathematics and Econometrics, Hunan University, Changsha, 410082, PR China.
E-mail addresses: chenting0715@126.com (T. Chen), lhhuang@csust.edu.cn (L. Huang), pyu@uwo.ca (P. Yu), huangwentao@163.com (W. Huang).

https://doi.org/10.1016/j.nonrwa.2017.10.003

completely solved for $n = 2$. For cubic systems, Yu and Han [4,5], Liu and Huang [6] proved $H(3) \geq 12$ by studying Hopf bifurcation. Later, Li et al. [7] constructed a Hamiltonian system and applied proper perturbations to prove $H(3) \geq 13$. On the other hand, Liu and Li [8] investigated the cyclicity problem for a Z_2-equivariant cubic system, and showed that this system can have 13 limit cycles, with a large-amplitude limit cycle at infinity, surrounding 12 small-amplitude limit cycles around two symmetric foci.

To completely study bifurcation of limit cycles in system (1), it is necessary to include studying the bifurcation of limit cycles at infinity. The bifurcation of limit cycles at infinity was studied by Shi [2] 30 years ago, and later the birth of a unique limit cycle at infinity is shown by Sotomayor [9]. In order to find maximal number of limit cycles bifurcating from infinity for cubic systems, Blows and Rousseau [10] computed the first five Lyapunov quantities at infinity for a class of cubic systems:

$$\begin{cases} \dot{x} =\lambda_1 x - \eta y + Ax^2 + (B+2D)xy + Cy^2 + \lambda_2 x(x^2+y^2) - y(x^2+y^2), \\ \dot{y} =\eta x - \lambda_1 y + Dx^2 + (E-2A)xy - Dy^2 + x(x^2+y^2) + \lambda_2 y(x^2+y^2), \end{cases} \tag{2}$$

and studied the limit cycles bifurcating from the origin and infinity. Liu and Chen [11] constructed an example of cubic system with 6 limit cycles bifurcating from infinity. Liu and Huang [12] proved that a cubic polynomial system can have 7 limit cycles near infinity. Actually, studying the bifurcation of limit cycles at infinity is quite similar to studying Hopf bifurcation at the origin, via a transformation based on Poincaré return map. However, a uniform upper bound of the number of limit cycles bifurcating at infinity for polynomial vector fields is still unknown.

Recently, increasing interest has been focused on bifurcation of limit cycles in discontinuous or non-differentiable, i.e., non-smooth dynamical systems. In this paper, we consider the piecewise polynomial system (or the so-called switching polynomial system) with a switching line on the x-axis, given in the form of

$$(\dot{x},\dot{y}) = \begin{cases} \left(\sum_{k=1}^{+\infty} X_k^+(x,y,\lambda), \sum_{k=1}^{+\infty} Y_k^+(x,y,\lambda)\right), & \text{for} \quad y>0, \\ \left(\sum_{k=1}^{+\infty} X_k^-(x,y,\lambda), \sum_{k=1}^{+\infty} Y_k^-(x,y,\lambda)\right), & \text{for} \quad y<0, \end{cases} \tag{3}$$

where $X_k^{\pm}(x,y,\lambda)$ and $Y_k^{\pm}(x,y,\lambda)$ are homogeneous polynomials of degree k in x and y , $\lambda \in \Lambda \subset \mathbf{R}^s$ is a parameter vector. System (3) includes two systems: the first one is called the upper system, defined for $y > 0$, and the second one is called the lower system, defined for $y < 0$.

The investigation of the more general piecewise systems, described by

$$(\dot{x},\dot{y}) = \begin{cases} (X^+(x,y), Y^+(x,y)), & \text{for} \quad y>0, \\ (X^-(x,y), Y^-(x,y)), & \text{for} \quad y<0, \end{cases} \tag{4}$$

started a half century ago [13–15]. Here, $X^{\pm}(x,y)$ and $Y^{\pm}(x,y)$ are real analytic functions in a neighborhood of the origin. Note that system (4) is usually considered as a differential system with discontinuous right sides, and simply called discontinuous system. Such systems can exhibit rich complex dynamical phenomena. Since the analytic functions $X^{\pm}(x,y)$ and $Y^{\pm}(x,y)$ in (4) can be expanded into the form of (3) with the coefficients treated as parameters, researchers generally consider them equivalent and use either one as they wish. Filippov established some basic qualitative theory in [15] for such discontinuous systems. In the study of analytic system (4), the cyclicity problem is fundamental in the qualitative analysis. Coll et al. [16] developed a method for computing the Lyapunov constants to study bifurcation of small-amplitude limit cycles. They derived the explicit formulas for computing the first three Lyapunov quantities. Let $P(n)$ denote the maximal number of limit cycles for system (3) of degree n. Gasull and Torregrosa [17] obtained