Optik - International Journal for Light and Electron Optics 169 (2018) 8–11

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Focusing of a radially polarized, cylindrically symmetric, vector beam by a thin lens: Simplified analysis of the radial-to-axial polarization transformation and determination of the electromagnetic fields at and near focus

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ARTICLE INFO

Keywords:
Physical optics
Electromagnetic optics
Polarization
Lenses

ABSTRACT

The radial-to-axial polarization transformation that accompanies the focusing of a cylindrically symmetric vector beam by a thin lens is analyzed by use of polarization ray tracing and wave optics. A simple expression for the axial electric field at focus is derived in terms of an integral of the incident radial electric field over the entrance pupil of the lens. Subsequently, from Maxwell's equations, the near-axis radial electric field and azimuthal magnetic field in the focal plane are determined. Finally, the concept of wave impedance is invoked to obtain the electric field on-axis near focus.

1. Introduction

Cylindrical vector beams, their focusing properties, and potential applications have been the subject of considerable recent interest [1–4]. For a cylindrically symmetric radially polarized (CSRP) beam, a striking feature is the radial-to-axial polarization transformation (RAPT) that accompanies the focusing of such beam by a lens. This phenomenon was first analyzed by Youngworth and Brown [1] based on earlier formulation by Richards and Wolf [5].

In this paper (Section 1) we use polarization ray tracing [6,7] to provide a simple and direct explanation of the RAPT of a CSRP beam that is focused by a thin lens. In Section 2 we use wave optics to derive an integral expression for the axial electric field at the focal point for a given distribution of the radial electric field incident on the lens.

In Section 3 the radial electric field and azimuthal magnetic field just off axis in the focal plane are determined in terms of the axial electric field at and near focus from Maxwell's equations. And by invoking the concept of wave impedance, a plane-wave-like expression for the near-focus, on-axis electric field is derived in Section 4. Section 5 gives a brief summary of this work.

2. Radial-to-axial polarization transformation by a thin lens

Fig. 1 shows the focusing of a CSRP monochromatic light beam centered on and traveling along the z axis by a convergent thin lens L with focal length f. The thin lens may be a conventional convex lens or a graded-index lens with uniform thickness.

In cylindrical coordinates (r, ϕ, z), the incident radial electric field just to the left of the lens (not exactly as shown in Fig. 1) takes the form $\overrightarrow{E_i} = E_r(r)\widehat{r}(\phi)$. $E_r(r)$ is a function of radial distance r and $\widehat{r}(\phi)$ is the $\phi-$ dependent radial unit vector given by $\widehat{r}(\phi) = (\cos\phi)\widehat{x} + (\sin\phi)\widehat{y}$, where $\widehat{x}$ and $\widehat{y}$ are the fixed unit vectors along the Cartesian x and y axes (in Fig. 1 the y axis is normal

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https://doi.org/10.1016/j.ijleo.2018.05.026
Received 20 March 2018; Accepted 8 May 2018
0030-4026/

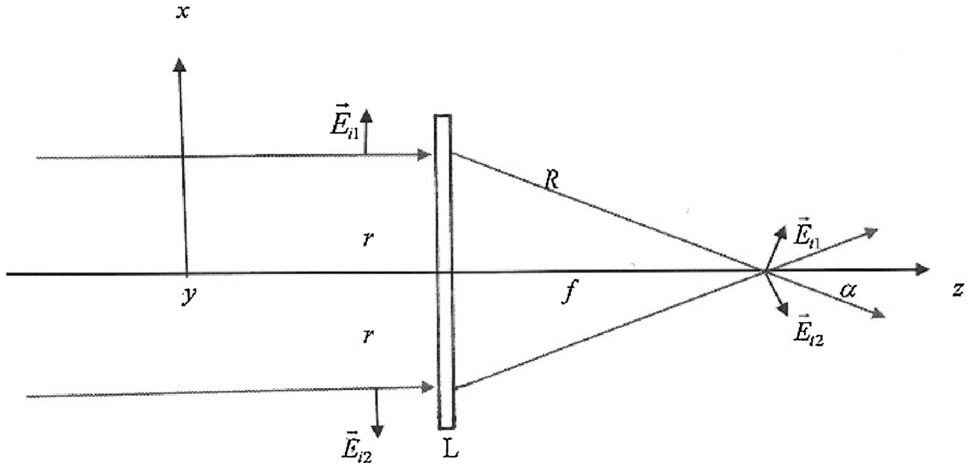

Fig. 1. Focusing radially polarized light beam by a thin lens.

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We assume that an incident ray parallel to the z axis is refracted in and out of the cylindrically symmetric lens in the same $\phi=$ constant meridional plane. And in any given such plane the linear polarization before and after the lens is the p polarization parallel to that plane.

In Fig. 1 consider a pair of parallel incident rays 1 and 2 at the same radial distance r above and below the z axis at azimuth angles $\phi = 0$ and $\phi = \pi$, respectively. The electric vectors associated with these two rays are denoted by $\overrightarrow{E_{i1}}$ and $\overrightarrow{E_{i2}}$, respectively. The corresponding electric vectors of the transmitted rays at the focal point of the lens are denoted by $\overrightarrow{E_{t1}}$ and $\overrightarrow{E_{t2}}$, respectively.

Because of cylindrical symmetry, it is apparent from Fig. 1 that the radial components of the electric vectors $\overrightarrow{E_{t1}}$ and $\overrightarrow{E_{t2}}$ at focus cancel each other out and their vector sum $\overrightarrow{E_{t1}} + \overrightarrow{E_{t2}}$ is oriented along the z axis. This is true for all such pairs of symmetric rays, hence it follows that the resultant electric field at focus is purely z-directed. This reasoning provides the simplest explanation of the RAPT phenomenon.

For an incident light ray at a given radial distance r from the z axis, the transmitted phasor electric field E_t at focus can be related to the incident phasor electric field E_i by

$$E_t = \exp(-jkR)\tilde{t}_L(r)E_i. \tag{1}$$

In Eq. (1) $R = (r^2 + f^2)^{1/2}$, $\tilde{t}_L(r)$ is the complex-amplitude transmittance of the lens [8],

$$\tilde{t}_L(r) = \tilde{t}_0 \exp(jkr^2/2f). \tag{2}$$

In Eq. (2) $\tilde{t}_0$ is the complex-amplitude transmittance of the lens at its center, $k = 2\pi/\lambda$, and λ is the wavelength of incident light. The projection of E_t along the z axis at focus is given by

$$E_{tz} = (\sin\alpha)E_t = (r/R)E_t, \tag{3}$$

and α is the angle that the refracted ray makes with the z axis. From Eqs. (1)–(3) we obtain

$$E_{tz} = \tilde{t}_0(r/R)\exp(-jkR)\exp(jkr^2/2f)E_i(r);$$
$$R = (r^2 + f^2)^{1/2}, \tag{4}$$

A differential axial electric field $dE_z(F)$ at focus, due to a circular ring of radius r and differential thickness dr in the input plane, is obtained by multiplying E_{tz} of Eq. (4) by the differential areal weighing factor

$$dw = 2\pi rdr/(\pi a^2) = 2rdr/a^2. \tag{5}$$

Subsequent integration over the illuminated circular entrance pupil of the lens of radius a gives the full axial field at focus as:

$$E_z(F) = (2\tilde{t}_0/a^2)\int_0^a (r^2/R)\exp(-jkR)\exp(jkr^2/2f)E_i(r)dr. \tag{6}$$

Eq. (6) provides a simple expression for the axial electric field at focus, obtained by straightforward application of ray and wave optics.

In Section 3 symmetry considerations and two of Maxwell's equations are used to relate the radial electric field $E_r(r)$ and azimuthal magnetic field $H_\phi(r)$ just off axis ($0 < r < < a$) in the focal plane to the axial electric field E_z at focus.