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Optical solitons with modified extended direct algebraic method for quadratic-cubic nonlinearity

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ABSTRACT

This paper implements modified direct algebraic method to secure soliton solutions in quadratic-cubic medium. Bright and dark-singular combo solitons are obtained along with their existence criteria. Several other solutions to the model including periodic singular solutions are also presented.

1. Introduction

There are several forms of nonlinear media that support propagation of soliton molecules across trans-continental and trans-oceanic distances. These include Kerr law, power law, dual-power law, triple-power law, log-law, exponential-law and many more. This paper investigates soliton dynamics in a quadratic-cubic (QC) medium. Various forms of integration architecture are available to study the dynamics of solitons and solitary waves across various disciplines [1–15]. These range from inverse scattering transform, traveling wave hypothesis, G'/G-expansion scheme, Kudryashov's method, the method

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of undetermined coefficients, extended trial equation scheme, Hirota's bilinear method, semi-inverse variational principle and several others. This paper applies the modified extended direct algebraic method to secure soliton and other solutions to the governing nonlinear Schrödinger's equation (NLSE) that is with QC nonlinearity. This integration scheme was first proposed by Soliman during 2008 and has been successfully implemented ever since to a variety of nonlinear evolution equations [11]. The following section introduces the governing model and the integration scheme along with its successful implementation to NLSE.

1.1. Governing equation

The governing NLSE in dimensionless form with QC nonlinearity reads [2]:

$$iq_t + aq_{xx} + \left(b_1|q| + b_2|q|^2\right) q = 0, \tag{1}$$

where the independent variables are x and t that represents spatial and temporal variables respectively. The dependent variable $q(x, t)$ gives the complex-valued wave profile and $i = \sqrt{-1}$. The coefficient of the real-valued constant a is group velocity dispersion (GVD). The two nonlinear terms are with coefficients b_1 and b_2 which are both real-valued constants. Solitons are the outcome of a delicate balance that exist between dispersion and nonlinearity.

1.2. Succinct review of modified extended direct algebraic method

We suppose that the given nonlinear partial differential equation for $u(x, t)$ to be in the form

$$P(u, u_t, u_x, u_{tx}, u_{tt}, u_{xx}\ldots) = 0, \tag{2}$$

where P is a polynomial in its arguments. The essence of the modified extended direct algebraic method can be presented in the following steps [1,5,9,11,12,14]:

Step 1: Seek traveling wave solutions of Eq. (2) by taking $u(x, t) = U(\xi)$, $\xi = kx - \omega t$ and transform Eq. (2) to the ordinary differential equation.

$$Q(U, U', U'', \ldots) = 0, \tag{3}$$

where primes denote the derivative with respect to ξ.

Step 2: We introduce the solution $U(\xi)$ of Eq. (3) in the form of a finite series as given by [5,1,12,11,9,14].

$$U(\xi) = \sum_{i=-N}^{N} a_i \phi(\xi)^i, \tag{4}$$

where a_i are real constants with $a_N \neq 0$ to be determined, N is a positive integer to be determined. $\phi(\xi)$ express the solution of the following equation [9]:

$$\phi'(\xi) = \sqrt{c_0 + c_1\phi(\xi) + c_2\phi^2(\xi) + c_3\phi^3(\xi) + c_4\phi^4(\xi) + c_5\phi^5(\xi) + c_6\phi^6(\xi)}, \tag{5}$$

where c_i are constants and can be discussed as in [15].

Step 3: Determine N. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in Eq. (3).

Step 4: Substituting Eq. (4) together with Eq. (5) into Eq. (3) yields an algebraic equation involving powers of $\phi(\xi)$. Equating the coefficients of each power of $\phi(\xi)$ to zero and discussing the value of c_i[15] gives a system of algebraic equations for a_i. Then, we solve the system with the aid of a computer algebra system (CAS), such as Mathematica or Maple, to determine these constants. On the other hand, depending on the value of parameters c_i[15], the solutions of Eq. (3) are well known to us. So, as a final step, we can obtain exact solutions of the given Eq. (1).

2. Application of the modified extended direct algebraic method

To start off with the method, the initial assumption of the solution structure of Eq. (1) is taken to be:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{6}$$

where

$$s = x - vt, \tag{7}$$

and the component ϕ is given by

$$\phi(x, t) = -\kappa x + \omega t + \theta. \tag{8}$$