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Technical note

A procedure for determining parameters of a simplified ligament model

Jeff M. Barrett, Jack P. Callaghan *,[1]

University of Waterloo, 200 University Avenue, Waterloo N2L 3G1, Ontario, Canada

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ABSTRACT

A previous mathematical model of ligament force-generation treated their behavior as a population of collagen fibres arranged in parallel. When damage was ignored in this model, an expression for ligament force in terms of the deflection, x, effective stiffness, k, mean collagen slack length, μ, and the standard deviation of slack lengths, σ, was obtained. We present a simple three-step method for determining the three model parameters (k, μ, and σ) from force-deflection data: (1) determine the equation of the line in the linear region of this curve, its slope is k and its x -intercept is $-\mu$; (2) interpolate the force-deflection data when x is $-\mu$ to obtain F_0; (3) calculate σ with the equation $\sigma = \sqrt{2\pi}F_0/k$.

Results from this method were in good agreement to those obtained from a least-squares procedure on experimental data – all falling within 6%. Therefore, parameters obtained using the proposed method provide a systematic way of reporting ligament parameters, or for obtaining an initial guess for nonlinear least-squares.

1. Introduction

Some ligaments, on a histological level, are composed of a population of collagen fibres arranged parallel to one-another (Chazal et al., 1985; Kumar, 2001). The primary function of ligaments is to maintain the structural integrity of joints by transmitting tensile forces between bones. A secondary role is to provide proprioceptive information (Johansson, 1991; Solomonow, 2004). Damage to ligaments alters the stability of joints, may cause pain, and may be a catalyst for chronic disorders like osteoarthritis (Kumar, 2001). Because of their pivotal role in healthy joint function, accurate models of their force-production behavior have far-reaching applications from improving larger scale biomechanical models to aiding foundational research that examines the function of ligaments.

Ligaments respond to tensile loading with a characteristic force-deflection curve, featuring a prominent toe region immediately followed by a linear region (Chazal et al., 1985; Frisén et al., 1969a, 1969b; Rigby et al., 1959). The toe region is attributed to the progressive recruitment of collagen fibres as they uncrimp in resisting the applied load (Franchi et al., 2007). As more fibres are gradually involved in providing tension, there is a corresponding gradual increase in the stiffness of the ligament. The stiffness saturates to a constant value once all the collagen fibres have been uncrimped. With continued stretching the fibres progressively break as their individual tolerance is exceeded. Eventually this can cumulate into complete ligament rupture. This mechanism has now been represented in a model (Barrett and Callaghan, 2017), which, when failure is ignored, encapsulates the toe and linear region behavior with Eq. (1).

$$F(x; k, \sigma, \mu) = \frac{k\sigma}{\sqrt{2\pi}} \exp\left(-\frac{(\mu + x)^2}{2\sigma^2}\right) + \frac{k(\mu + x)}{2}\left[\operatorname{erf}\left(\frac{\mu + x}{\sqrt{2}\sigma}\right) + 1\right] \tag{1}$$

Here, F is the force produced by a ligament and x is its elongation. There are three parameters: k, the ligaments effective stiffness; σ, the standard deviation of collagen slack lengths; μ the average collagen slack length; and $\operatorname{erf}(\cdot)$ is the error-function. Here we present a straightforward method for determining these parameters from experimental data. In addition, we attempt to disseminate the implications each model parameter in Eq. (1) has on the resulting shape of the force-deflection curve.

2. Methods

2.1. A stepwise parameter finding technique

With an increase in x, Eq. (1) is asymptotically equivalent to the equation of a line with slope k and intercept μk (Eq. (2)). This is

* Corresponding author.
E-mail addresses: jeffery.barrett@uwaterloo.ca (J.M. Barrett), jeffery.barrett@uwaterloo.ca (J.P. Callaghan).
[1] ASME Membership:101987826

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Nomenclature

F	Force
x	Displacement from neutral
μ	Average slack length in the population of collagen fibres.
σ	Standard-deviation of slack lengths in the population of collagen fibres.

because the first term tends to zero, while the term in the square brackets tends to two. This coincides with the linear region from the model, and thus, measuring the slope of the ligament's linear region is equivalent to k.

$$\ell(x) = k(\mu + x) \quad (2)$$

This equation has an x -intercept at $x = -\mu$, thus, extending this line to the x -axis gives the value for μ. Denoting the x -intercept as x_0, with corresponding force-value of F_0. Substituting $x = x_0 = -\mu$ into Eq. (1), the second term becomes zero and the exponential in the first term becomes unity. This yields a relationship between σ, k and F_0 which can be rearranged for σ:

$$\sigma = \frac{\sqrt{2\pi}\,F_0}{k} \quad (3)$$

where F_0, in this investigation, was obtained using third-order spline interpolation on the original force-deflection curve. This process is graphically illustrated in Fig. 1.

This method was applied to the force-deflection data from Mattucci (2011) for the anterior longitudinal ligament of the middle cervical spine (C4-C6) of males loaded at a low strain-rate (0.5 / s). Results using this stepwise procedure were compared to a non-linear least-squares method fitted to the data provided by the scipy python package. The initial guess for the fitting algorithm were the parameters identified through this stepwise procedure. The linear region was pre-determined by Mattucci (2011) using the methods of Chandrashekar et al. (2008).

2.2. Sensitivity analysis and compliance function from Eq. (1)

To perform a sensitivity analysis on Eq. (1), its partial derivative was taken with respect to each parameter (k, μ, σ), and evaluated at the parameter values obtained using the stepwise method (c.f. Cashaback et al., 2013). We plotted these partial derivatives over a range of displacements to determine which regions were more sensitive to changes in which parameters. Additionally, we increased and decreased each parameter value by 10%, independently, and noted the corresponding change in the resulting force-deflection curve to provide further analysis on these parameters.

Calculating tendon compliance is a crucial step in solving the differential equations that govern the Hill-type muscle model (Zajac, 1989). As tendons and ligaments are similar histologically, it stands to reason that Equation (1) may also be useful as a model of tendon force. We have included the compliance function calculated from Eq. (1) in Appendix B.

3. Results

The results obtained from this simplified method compare very well to using a non-linear least-squares optimization (Table 1), and the resulting curves from the two methods agree very well with experimental data (Fig. 2).

Perturbations in each parameter resulted in changes to the force-deflection curve (Fig. 3). Of note, the effects of σ perturbations are quite small, indicating that it takes a fairly substantial change in σ to induce noticeable changes in force-deflection behaviour. Unaltered partial derivatives are calculated and presented in Appendix A.

4. Discussion

The proposed method, along with a rigorous sensitivity analysis, clarifies how the parameters in Eq. (1) interact with one-another in relation to the force-elongation curve. For instance, the effective collagen stiffness, k, is the slope of the linear region of the force-deflection curve (Fig. 4). Coincidentally, this variable is exactly what has been reported in several previous ligament studies (Chandrashekar et al., 2008; Chazal et al., 1985; Trajkovski et al., 2014; Yoganandan et al., 1989). The parameter

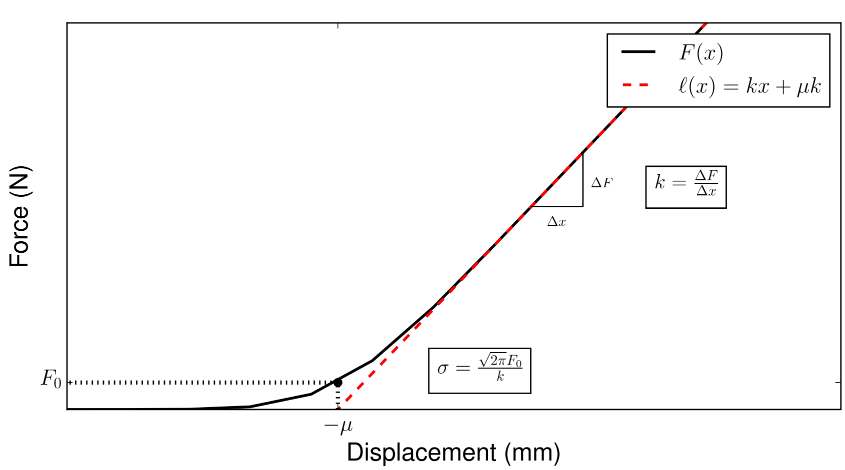

Fig. 1. A graphical representation of the procedure for obtaining model parameters. Starting with a Force-Deflection Curve, obtained from experimental data ($F(x)$), continuing the line from the linear region to the x -axis provides - μ. Taking a vertical line from this point up to the original Force-Deflection curve gives the value $F_0 = F(-\mu)$, which can be used to calculate σ, with k being the slope of the linear region of the original force displacement data.

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