Journal of Economic Behavior and Organization 149 (2018) 123–136

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

Multi-player race

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ARTICLE INFO

Article history:
Received 20 November 2017
Revised 8 March 2018
Accepted 10 March 2018

JEL classification:
C72
D72
D74

Keywords:
Contests
Discouragement effect
Dynamic games
Momentum effect
Race

ABSTRACT

We present a model of race with multiple players and study players' effort choices and expected prizes in equilibrium. We show that, in equilibrium, once any two players win one battle each, the remaining players do not exert any effort anymore. This turns the continuation game into a two-player race. This is different than the results in previous two-player models of race, which report that all states of the game are reached with positive probabilities. We also provide a set of comparative static results on the effects of the number of players and the victory threshold.

1. Introduction

Multi-battle competitive interactions are ubiquitous. Some obvious examples are sports tournaments, patent races, election campaigns, public debates, wars, etc. The contest theory literature produced various models of multi-battle dynamic contests, such as race, tug-of-war, elimination tournaments, war of attrition, and repeated incumbency fights (see Konrad, 2012; Vojnovic, 2016). This paper studies a model of *race*. For almost all examples mentioned above, it would be natural to consider a multi-player interaction. Interestingly, earlier work on race exclusively focused on two-player interactions. Here, we study a multi-player race.

In a race,[1] players simultaneously decide on the efforts they exert in each battle and the player who first accumulates a certain number of battle victories wins the contest. We study a multi-player game of race, where (i) the battle outcomes are determined with a Tullock contest success function (CSF), (ii) there is no intermediate prize, (iii) no losing punishment, and (iv) no discounting. Players are symmetric in that they have identical winning prizes and cost functions, and the victory threshold (i.e., the number of victories needed to win the contest) is the same for each player. We describe the differences and similarities between our model and the earlier work on race in detail, in Section 2.

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[1] We adopt the definition put forward by Konrad and Kovenock (2009).

https://doi.org/10.1016/j.jebo.2018.03.008

Table 1
Some recent studies on two-player races and Colonel Blotto games.

	Tullock CSF	All-pay CSF
Dynamic Models	**Theoretical:** Baba (2012); Harris and Vickers (1987); Klumpp and Polborn (2006) **Experimental:** Baba (2012); Mago et al. (2013); Irfanoglu et al. (2015)	**Theoretical:** Gelder (2014); Konrad and Kovenock (2009) **Experimental:** Deck and Sheremeta (2012); Mago and Sheremeta (2017)
Static Models	**Theoretical:** Klumpp and Polborn (2006); Robson (2005) **Experimental:** Chowdhury et al. (2013); Duffy and Matros (2017); Irfanoglu et al. (2015)	**Theoretical:** Hart (2008); Kvasov (2007); Roberson (2006) **Experimental:** Chowdhury et al. (2013); Mago and Sheremeta (2017)

We show that, in equilibrium, once any two players win one battle each, all of the other players are totally discouraged, i.e., they do not exert any effort in the continuation game, which turns the continuation game into a (symmetric or asymmetric) two-player race. This finding is different than the earlier theoretical works on two-player race (e.g., Gelder, 2014; Klumpp and Polborn, 2006; Konrad and Kovenock, 2009), which documents *pervasiveness* (i.e., all states of the game are reached with positive probabilities).

This result shares a flavor similar to the well-known Duverger's Law in the theory of elections, which states that plurality voting tends to favor a two-party system (Duverger, 1972). Race is similar to plurality voting in that it does not impose any requirement on the winning margin but only imposes an absolute (or, nominal) victory threshold. We show that race produces a two-player contest on the equilibrium path, which can be read as a strategic foundation for the contest theory version of the Duverger's Law. In addition to that, if one entertains an application of our model to elections (as in Klumpp and Polborn, 2006), our main result could also be interpreted as a strategic support for elections with runoffs, where only two candidates qualify for the second round.

We also conduct comparative static analyses on the number of players and the victory threshold to see their effect on important markers of dynamic contests such as the *discouragement effect* (decrease in the efforts of laggards), *rent dissipation* (the ratio of exerted effort to the prize value), and the *momentum effect* (increased chances of the player who won the first battle for winning the second battle or the whole contest). The results can be summarized as follows: (i) The aforementioned extreme form of discouragement is present independent of the number of players as long as this number is greater than two. (ii) In a model with a victory threshold of two, an increase in the number of players increases rent dissipation and tempers the momentum effect. (iii) Interestingly, in a model with three players, an increase in the victory threshold first decreases then increases rent dissipation. The converse is true for the momentum effect. A victory threshold of four minimizes rent dissipation and maximizes the momentum effect.

The organization of the paper is as follows. In Section 2, we present an overview of the related literature. In Section 3, we first present our results for the case of three-player race ($n = 3$) with a victory threshold of two ($t = 2$). Later, we extend our results to the models with more than three players (keeping $t = 2$) and with a victory threshold of more than two (keeping $n = 3$). Section 4 concludes.

2. Literature review

The literature on dynamic contests is large (see Konrad, 2012; Vojnovic, 2016). Here, we primarily focus on earlier works on *race*. The first paper to formally study race as a dynamic multi-stage contest (with simultaneous actions in each stage) is Harris and Vickers (1987).[2] These authors model a patent race between two firms as a multi-stage contest with sequential battles. In each stage (or, battle), firms simultaneously spend efforts/resources in research and development; and the battle outcome is a stochastic function of their efforts. The first firm to reach a certain number of stage victories wins the patent race. The authors show that the leader spends greater effort than the follower does, and these efforts increase as the gap between the two firms decreases.

Our paper is most closely related to Klumpp and Polborn (2006), who study the primaries, i.e., sequential elections in single states in the U.S., as a two-player race. The authors compare two alternative temporal structures (dynamic vs. static) on the basis of the prevalence of the momentum effect, the expected costs, and the candidates' winning probabilities. Using a Tullock CSF to determine the outcome of individual elections, they show that players spend relatively large amounts of resources in the first battle and that winning the first battle creates a *momentum*. The discouragement is not extreme in the sense that the laggard keeps investing non-zero amounts.

[2] Harris and Vickers (1985) also study a race, but in their model players take actions sequentially, which does not fit the definition we adopt here.