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# Uncertainty evaluation of multivariate quantities: A case study on electrical impedance

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A B S T R A C T

The paper discusses the evaluation of the uncertainty of a multivariate quantity using the Law of Propagation of Uncertainty defined in the Guide to the Expression of Uncertainty in Measurement (GUM) and a Monte Carlo method according to the GUM's Supplement 2. The quantity analysed is the electrical impedance, which is not a scalar but a complex quantity. The used measuring method allows the evaluation of the impedance and of its uncertainty in different ways and the corresponding results are presented, compared and discussed. For comparison purposes, results of the impedance uncertainty obtained using the NIST Uncertainty Machine are also presented.

## 1. Introduction

The publication, in 1993, of the first version of the Guide to the Expression of Uncertainty in Measurement (GUM) was the significant result of the work of more than 15 years developed by individuals and organizations to produce an internationally accepted procedure for expressing measurement uncertainty and for combining individual input uncertainty components into a single total uncertainty for more complex measurement models. The Guide, revised in 1995 and 2008 [1], materializes Recommendation INC-1 (1980) by the CPIM Working Group on the Statement of Uncertainties by providing a conceptual framework allowing a consistent treatment of uncertainty contributions for all to use. Complemented by the contribution of many other publications [2–7], the impact of GUM in Metrology increased when, in 2005, standard ISO/IEC17025 [8] established that accredited test laboratories were required to estimate the uncertainty and to report it. The core of the GUM is the assignment of uncertainties to each measurement model input and the use of the Law of Propagation of Uncertainty (LPU) to estimate the output measurement uncertainty (called combined uncertainty because it combines the input uncertainties and the characteristics of the measurement model). The LPU is widely considered the main tool for assessing uncertainty in measurement models with multiple inputs. However, the LPU has some shortcomings, namely: not dealing with non-symmetrical measurement uncertainty distributions, not dealing with non-linearity of the measuring system, and mainly considering only models having a single scalar output quantity.

To cope with these limitations, one possibility is the use of the Monte Carlo method (MCM) which has been analysed by many authors (e.g., [9–18]) and addressed in two supplements to the GUM – Supplement 1 and Supplement 2. Based on the same basic ideas of the GUM – measurement model and uncertainty of the input quantities – Supplement 1 [19] deals also only with models having a single scalar output quantity, introduces numerical

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simulation using the MCM as an alternative to analytical or numerical integration for calculating the combination of the probability distributions of the input quantities involved in the propagation of distributions required for the evaluation of the output measurement uncertainty. Supplement 2 [20] extends the use of GUM and Supplement 1 to multivariate measurement models, i.e., models with more than one output quantity. It is true that *the GUM includes examples, from electrical metrology, with three output quantities [JCGM 100:2008 H.2], and thermal metrology, with two output quantities [JCGM 100:2008 H.3]* [20], but Supplement 2 *describes a generalization of that Monte Carlo method to obtain a discrete representation of the joint probability distribution for the output quantities of a multivariate model. The discrete representation is then used to provide estimates of the output quantities, and standard uncertainties and covariances associated with those estimates* [20].

Supplements 1 and 2 to the GUM originate some inconsistency between the GUM and the Supplements [21] (e.g., Supplement 1 does not require classification into Type A and Type B evaluation of uncertainties). This fact, added to some non-solved limitations of the GUM itself, led to the need of a revision of the GUM that is underway [21]. As far as it is known, two documents have been prepared by the Working Group 1 of the Joint Committee for Guides in Metrology (JCGM), concerning the Guide to the Expression of Uncertainty in Measurement. These will circulate in 2015 for review through the eight JCGM member organizations (BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML). Perhaps the most expected and significant change will be the assertion of the Bayesian view of probability in the evaluation of measurement uncertainty. The available knowledge is then used to deduce a PDF for the quantity concerned, and the standard deviation of this PDF is used as the standard uncertainty.

Evaluation of the uncertainty of multivariate quantities is usually a challenging and disputable problem namely when the quantity can be measured and expressed in different ways. Such is the case of the electrical impedance. Supplement 2 of GUM includes, in its paragraph 9.4 – Simultaneous measurement of resistance and reactance, an example of evaluation of the uncertainty of electrical impedance. In the next paragraphs the subject is revisited and results obtained using a possible measuring method of both the expression of the impedance value and of the evaluation of its uncertainty are presented and discussed.

## 2. Electrical impedance

Electrical impedance extends the concept of resistance to alternating current (AC) circuits, describing not only the relative amplitudes of the voltage and current, but also their phase difference. Electrical impedance measurement is important not only in the analysis of electric circuits but also for other purposes. In fact, because the electrical impedance allows the quantification of the behaviour of a conducting medium to an electric current, the impedance measurement is used in the determination of the electromagnetic properties of materials and is the basis of various methods of electrical transduction with applications in fields such as chemistry [22,23] and biomedicine [24–27].

Sometimes abusively used in other time varying regimes, electrical impedance, usually represented by $Z$, is an electric quantity defined in the context of sine wave alternating current. Thus if $v(t)$ is the impedance sinusoidal voltage with constant amplitude $V_M$, constant frequency $f$, and constant initial phase $\varphi_v$

$$v(t) = V_M \cos(2\pi f t + \varphi_v) = \mathrm{Re}(V_M e^{j2\pi f t} e^{j\varphi_v}), \tag{1}$$

where $\mathrm{Re}(x)$ is the real part of $x$, the impedance current is

$$i(t) = I_M \cos(2\pi f t + \varphi_i) = \mathrm{Re}(I_M e^{j2\pi f t} e^{j\varphi_i}). \tag{2}$$

In complex amplitudes (phasors) both can be written as

$$V = V_M e^{j\varphi_v} \text{ and } I = I_M e^{j\varphi_i}. \tag{3}$$

The impedance is not a phasor but a complex number defined as the ratio $V/I$ whose amplitude (module) is $|Z|$ and whose phase is $\varphi$

$$\begin{aligned} Z &= \frac{V}{I} = \frac{V_M}{I_M} e^{j(\varphi_v - \varphi_i)} = \frac{V_M}{I_M} e^{j\varphi} = |Z|\, e^{j\varphi} \\ &= |Z| \cos(\varphi) + j|Z| \sin(\varphi). \end{aligned} \tag{4}$$

The principles, methods, equipment and procedures for electrical impedance measurement are diverse [28] depending, namely, on the frequency and on the application. In the following sections, and because the purpose here is to compare the uncertainty evaluation using two different methods, the considered case is that of electrical impedance measurements based on digital acquisition of sinusoidal voltages and estimation of their amplitudes and phase difference, as described in [29].

## 3. Measurement method

Fig. 1 depicts the setup for the measurement of impedance $Z$ through the measurement of the voltages at its terminals and the terminals of a reference impedance $Z_R$. These voltages are buffered with two instrumentation amplifiers (IAs) with unitary gain and simultaneously acquired using two analogue-to-digital converters (ADCs). The series of $Z$ and $Z_R$ is excited by the output of a sine wave generator that defines the measurement frequency. Fig. 2 shows an example of the sampled signals for a 1 kHz measurement frequency, sampled at 48 kS/s. The

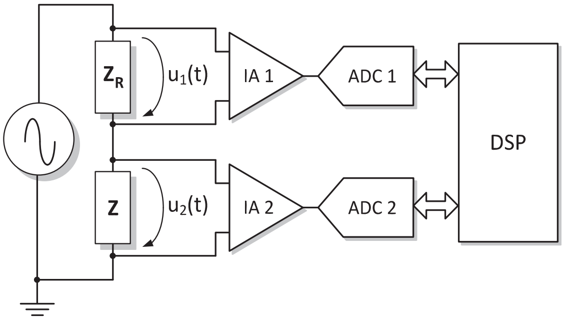

**Fig. 1.** Setup for the measurement of $Z$.