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Developmental foundations of children's fraction magnitude knowledge

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A B S T R A C T

The conceptual insight that fractions represent magnitudes is a critical yet daunting step in children's mathematical development, and knowledge of fraction magnitudes influences children's later mathematics learning including algebra. In this study, longitudinal data were analyzed to identify the mathematical knowledge and domain-general competencies that predicted 8th and 9th graders' (n = 122) knowledge of fraction magnitudes and its cross-grade gains. Performance on the fraction magnitude measures predicted 9th grade algebra achievement. Understanding and fluently identifying the numerator-denominator relation in 7th grade emerged as the key predictor of later fraction magnitudes knowledge in both 8th and 9th grades. Competence at using fraction procedures, knowledge of whole number magnitudes, and the central executive contributed to 9th but not 8th graders' fraction magnitude knowledge, and knowledge of whole number magnitude contributed to cross-grade gains. The key results suggest fluent processing of numerator-denominator relations presages students' understanding of fractions as magnitudes and that the integration of whole number and fraction magnitudes occurs gradually.

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1. Introduction

Children's understanding of fractions is central to their mathematical development (National Mathematics Advisory Panel, 2008; NMAP). It is a critical step in extending their understanding of numbers from whole numbers to other rational numbers, and is foundational to advanced mathematics learning including algebra, calculus, and statistics (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; NMAP, 2008; Siegler et al., 2012). Moreover, competence with arithmetic in general, which includes fractions, contributes to employment and wage opportunities in adulthood (Bynner, 1997). Unfortunately, fractions are one of the most challenging mathematical concepts for children to learn (Hoffer, Venkataraman, Hedberg, & Shagle, 2007; Ni & Zhou, 2005; NMAP, 2008). Students in the U.S. typically begin to learn fractions before 4th grade, but about half of 8th graders still cannot order the quantities represented by three fractions, and many of them cannot correctly answer that the sum of 12/13 and 7/8 is closer to 2 than to 19 or 21 (see also Bailey, Siegler, & Geary, 2014; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Hecht and Vagi, 2010; Mazzocco & Devlin, 2008; Siegler and Pyke, 2013; Siegler, Thompson, & Schneider, 2011). A nationally-representative survey of high school algebra teachers indicated that students'

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poor understanding of fractions and rational numbers more generally was a critical impediment to learning algebra (NMAP, 2008).

A more complete understanding of the prior knowledge and cognitive factors that contribute to fractions learning has the potential to inform instructional and remedial approaches and through this improve the long-term prospects of struggling students (Ni & Zhou, 2005; Siegler et al., 2011; Vosniadou, 2014). Many previous studies have focused on young children's natural number bias, that is, the ways in which their conceptual understanding of whole numbers (e.g., each number has a successor) interferes with their learning of rational numbers (e.g., Ni & Zhou, 2005; Van Dooren, Lehtinen, & Verschaffel, 2015). Children's emerging competence with rational numbers requires an inhibition of conflicting whole number concepts (e.g., multiplication of two numbers results in a larger number) (Van Hoof, Janssen, Verschaffel, & Van Dooren, 2015), and the insight that whole numbers and rational numbers also share a common, conceptual feature – they represent numerical magnitudes (Siegler et al., 2011). Our focus is on this latter aspect of competence with rational numbers, fractions in particular, and on the grades in which this and any associated deficits will directly undermine the learning of algebra (NMAP, 2008). Using data from a comprehensive longitudinal study of children's mathematical development, we show that 8th and 9th graders' understanding of how fractions are situated on the number line predicts 9th grade algebra achievement and identify the foundational mathematical knowledge and domain-general competencies that contribute to their competence with the fractions number line.

1.1. Foundational knowledge

Children's fractions competencies can be categorized into conceptual and procedural knowledge (Bailey et al., 2014, 2015; Byrnes & Wasik, 1991; Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001). Conceptual knowledge of fractions refers to the understanding of the nature and mathematical properties of fractions, including that a fraction consists of a numerator and a denominator, its magnitude is determined by the numerator-denominator relation, and that these magnitudes can be ordered on a number line (Hecht & Vagi, 2010; Jordan et al., 2013; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler et al., 2011; Vamvakoussi & Vosniadou, 2004). Procedural knowledge, in contrast, refers to understanding of the arithmetic algorithms that can be applied to fractions and their accurate use during problem solving (Bailey et al., 2015; Byrnes & Wasik, 1991). Both conceptual and procedural knowledge are essential to competence with fractions, and their development may be mutually reinforcing (Bailey et al., 2015; Rittle-Johnson et al., 2001; Siegler et al., 2011), although some children become competent with procedures before they conceptually understand fractions and vice versa (Hallett, Nunes, & Bryant, 2010).

As noted, with a recent integrated model of numerical development, Siegler and colleagues state that children's central conceptual insight occurs when they understand that fractions, as with whole numbers, represent magnitudes (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014). Identifying the knowledge foundational to this insight will facilitate the development of efficient approaches for fractions instruction. This knowledge likely includes children's understanding of the relation between the numerator and denominator. A poor understanding of this relation is common and reflected in children's treatment of the numerator and the denominator as independent whole numbers, one aspect of the whole number bias (Gelman & Williams, 1998; Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2004, 2010). Notably, although the understanding of the numerator-denominator relation and the understanding of fractions magnitudes are conceptually related, they are not identical. The former precedes and does not necessarily involve the latter. Taking fraction comparison as an example, one can judge 1/3 is numerically larger than 1/4 without necessarily representing the magnitude of each of the two fractions or being able to situate their relative position on the number line, using a simple heuristic: "when the numerator is given, the fraction value negatively varies with the denominator". Procedural knowledge of fractions may also be foundational for understanding fractions magnitudes because correct use of fraction arithmetic provides information about magnitude (e.g., $1/4 + 1/4 = 1/2$, so $1/2 > 1/4$) that may contribute to children's conceptual understanding of fractions (Bailey et al., 2015; Rittle-Johnson et al., 2001).

A mature understanding of fractions emerges as children integrate their understanding of fraction magnitudes with their understanding of whole number magnitudes (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014; Torbeyns, Schneider, Xin, & Siegler, 2015). Children with a strong grasp of whole numbers should then be better situated to make this integration than other children. Indeed, children who have a strong understanding of the magnitudes of whole numbers are more likely than other children to understand fractions as magnitudes, controlling for other numerical knowledge and general cognitive competencies, such as IQ (Bailey et al., 2012; Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013; Siegler & Pyke, 2013; Vukovic et al., 2014).

1.2. Current study

Our overarching goal was to explore prior mathematics knowledge and cognitive factors that contribute to children's understanding of how fractions are represented on the mathematical number line and cross-grade gains in this understanding. Among these factors, we particularly tested the hypothesis that children's understanding of the relation between numerators and denominators, as well as whole numbers as magnitudes contribute to their understanding of fractions magnitudes in the context of the number line. We focused on the latter as the outcome of interest because the number line itself is an important foundational aspect of mathematics and integrating knowledge of fractions magnitudes with their

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