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Local geoid determination in strip area projects by using polynomials, least-squares collocation and radial basis functions

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ABSTRACT

Orthometric heights are used in many engineering projects. However, the heights determined by the widely-used Global Navigation Satellite System (GNSS) are ellipsoid heights. Leveling measurements conducted with the purpose of determining the orthometric heights on points are quite arduous and time-consuming processes. To be able to use ellipsoid heights in engineering projects, their transformation to orthometric heights defined in the height datum of the region is necessary. Therefore, in terms of convenience and feasibility GNSS/levelling method is preferred in determining geoid heights. This method is based on the principle of transformation of ellipsoid heights to orthometric heights. In effect, the main purpose of the method can also be regarded as the estimation of geoid undulation values for the study area.

During the estimation process, polynomial (surface, curve) models are generally used. Polynomial models produce meaningful results for points which are scattered uniformly on the study area. However, for strip areas where the points scatter along a route (road etc. projects), the accuracy of the geoid heights obtained from these models is low. Therefore, different estimation techniques have to be implemented in strip areas instead of polynomial models. In this study, interpolation methods used in determining the geoid undulation of a strip area were researched and the identification of the best suitable method for the area was examined. For this purpose, geoid undulation values were calculated with the help of least-squares collocation (LSC) and radial basis functions such as Multiquadric (MQ), Thin Plate Spline (TPS) along with polynomial models, and results were presented. According to the results, it was observed that TPS, MQ, LSC methods respectively yield better results compared to polynomial methods.

1. Introduction

In recent years GNSS has been used frequently in many engineering applications such as surveying, geodesy, geophysics, navigation. One of the application areas of GNSS is the determination of point heights. In GNSS technique, point positions are defined as geocentric (X,Y,Z) based on WGS84 ellipsoid or as geodetic latitude, longitude and ellipsoidal heights (φ, λ, h) [1–3].

In many GNSS applications, users need the transformation between the ellipsoidal height and orthometric height [4–9]. The primary reason is the utilization of orthometric heights determined by leveling in engineering applications and the difficulty of conducting leveling measurements for each point. For this type of applications, geoid models with high accuracy are needed. To provide this, the precise estimation of geoid height (undulation) values produced with the basis of GNSS/levelling measurements is necessary.

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During estimation, polynomial (surface, curve) models are generally used. To generate a more precise geoid model, different estimation types such as least-squares collocation (LSC) [10,11], artificial neural networks [10,12], fuzzy logic [13], and radial basis functions (Multiquadric-MQ, Thin Plate Spline-TPS) [1] can be drawn upon.

In the GNSS/levelling method, existence of uniformly and homogeneously scattered points on the study area generally increases the quality of the estimation. However, for strip area projects such as highways, railways, and canals it is out of question. On the contrary, points vary along a route. This fact makes the production of geoid height information quite difficult [14]. In this type of strip area projects, calculating the geoid heights with LSC or radial basis functions plays an effective role on the results.

The aim of this study is to establish the inadequacy of polynomial methods in the transformation between ellipsoidal and orthometric heights especially in strip area projects and the attainment of geoid heights as sensitively as possible. Moreover, performances of polynomial, LSC, MQ and TPS models used in geoid undulation calculation were evaluated. For this purpose, geoid undulations were obtained by using the data of Nurdagi–Gaziantep highway project with the help of LSC and radial basis functions and performance indicators were determined. In addition, suggestions were given about choosing the suitable methods in the determination of geoid undulations for strip area projects.

2. Material and method

2.1. Polynomial interpolation

Interpolation technique with polynomial surfaces is widely used especially in determining local geoid heights with the GNSS/levelling method. The main purpose of this technique is based on the expression of the studied area with only one function. Moreover, polynomial surfaces can be preferred as trend surface in the implementations of interpolation methods such as kriging, multiquadric and collocation.

To be able to implement the method, reference points in sufficient number and scatter, of which orthometric (H) and ellipsoidal height (h) are known, are needed. With the help of these points, local geoid model is formulated [15,16]. Taking the size of the study area and the number of reference points into consideration, Eqs. (1) and (2) can be written for the curve and surface polynomial [17]

$$N(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots \quad \text{(Curve)} \tag{1}$$

$$N(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{n} a_{ij} x^i y^j = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + \ldots \quad \text{(Surface)} \tag{2}$$

Here, $i + j \leqslant n$, x and y indicate the coordinates of reference points, a_i and a_{ij} denote polynomial coefficients and n indicates the degree of the polynomial. In order to generate the geoid model at reference stations, geoid heights are calculated by Eq. (3) [6,16,18].

$$N = h - H \tag{3}$$

Geoid heights are defined as a function of the unknowns in accordance with the curve or surface model. The unknown coefficients of the polynomial (a_i or a_{ij}) are determined in accordance with the principle of Least-squares estimation (LSE) method,

$$\mathbf{x} = (\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{l}) \tag{4}$$

where $\mathbf{A}$ is the matrix of coefficients (design matrix), $\mathbf{P}$ is the weight matrix and $\mathbf{l}$ is the observation vector [19]. In this study it was postulated that weight of measurements (geoid heights) are equal. Residuals to be made to measurements and the standard deviation of the unit-weighted measurement are calculated using Eqs. (5) and (6), at the end of LSE solution.

$$\mathbf{v} = \mathbf{A}\mathbf{x} - \mathbf{l}; \quad \mathbf{P} = \mathrm{diag}(P_1, P_2, \ldots, P_n) \tag{5}$$

$$m_0 = \sqrt{\frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{n-u}} \tag{6}$$

Here, n indicates the number of measurement (observation), u indicates the number of unknown parameters, $\mathbf{v}$ denotes the residual vector of the measurements and $\mathbf{Ax}$ represents the deterministic part or trend. After the formulation of the curve or surface model, geoid heights are estimated for the unknown points. Firstly, the matrix of coefficients ($\mathbf{A}_p$) is formulated for the unknown points and the estimation results are obtained by Eq. (7),

$$\mathbf{l}_p = \mathbf{A}_p\mathbf{x} \tag{7}$$

where p is the number of points of which geoid heights are to be estimated. In local geoid determination studies, it is difficult to estimate the best suitable surface or curve polynomial degree for the study area at first glance. The best suitable polynomial degree for the area is usually determined according to the trial and error method. To this end, adjustment results and posteriori variance values are compared by starting with the first degree. From a theoretical perspective, the posteriori variance value is expected to decrease as the degree of the polynomial increases [20–22]. However, the increase of the degree of the polynomial may also lead to the increase of model errors. Therefore, one lower degree of the polynomial in which the model error starts to increase is accepted as the most suitable degree for the polynomial.

2.2. Least-squares collocation method (LSC)

In the general sense collocation is a mathematical model in which unknown parameters and interpolation problems are solved together. LSC method has been used and applied successfully in the geodetic problems such as physical geodesy, geoid determination and satellite mission applications [23–25]. The most important