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Fusion of multi-agent preference orderings in an ordinal semi-democratic decision-making framework

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ABSTRACT

This paper focuses on the problem of combining multi-agent preference orderings of different alternatives into a single fused ordering, when the agents' importance is expressed through a rank-ordering and not a set of weights. An enhanced version of the algorithm proposed by Yager (2001) is presented. The main advantages of the new algorithm are that: (i) it better reflects the multi-agent preference orderings and (ii) it is more versatile, since it admits preference orderings with omitted or incomparable alternatives. The description of the new algorithm is supported by a realistic example.

1. Introduction

A general problem, which may concern practical contexts of different nature, is to aggregate multi-agent orderings of different alternatives into a single fused ordering. Let us assume that there are M decision-making agents $D_1, D_2, \ldots, D_M$, each of which defines an ordering of n alternatives a, b, c, etc. This decision-making problem is fairly general [1–3] and can be applied to a variety of real-life contexts, ranging from *multi-criteria decision aiding* [4] to *social choice* [5,6] and *voting theory* [7,8].

The problem becomes more specific if the importance hierarchy of agents is expressed through a rank-ordering and not a set of weights defined on a *ratio* scale. This decision-making framework can be denominated as "ordinal semi-democratic"; the adjective "semi-democratic" indicates that agents do not necessarily have the same importance, while "ordinal" indicates that their hierarchy is defined by a crude ordering. The set of the possible solutions to the problem may range between the two extremes of (i) *full dictatorship*—in which the fused ordering coincides with the preference ordering by the most important agent (dictator)—and (ii) *full democracy*—where all agents' orderings are considered as equi-important.

Some years ago, Yager [9] proposed an algorithm to address the problem of interest in a relatively simple, fast and automatable way. Unfortunately, this algorithm (hereafter abbreviated as YA, which stands for "Yager's Algorithm") has two major limitations: (i) the resulting fused ordering may sometimes not reflect the preference ordering for the majority of agents and (ii) it is applicable to *linear* orderings only, without incomparabilities and omissions of the alternatives of interest. For details, we refer the reader to [9,10].

The objective of this paper is to enhance the YA so as to overcome its limitations and adapt to less stringent preference orderings. A new algorithm, denominated as "Enhanced (Yager's) Algorithm" (hereafter abbreviated as EYA), will be proposed.

The remainder of the paper is organized into two sections. Section 2 illustrates the EYA by presenting a realistic example. Section 3 summarizes the original contributions of the paper and its practical implications, limitations and suggestions for future research.

2. Enhanced Yager's Algorithm (EYA)

The EYA can be decomposed in three phases, which are individually described in the following sub-sections:

- Construction, normalization and reorganization of preference vectors.
- Definition of the reading sequence.
- Construction of the fused ordering.

2.1. Construction, normalization and reorganization of preference vectors

The YA is applicable to *linear* orderings only, where no alternatives are omitted and any two alternatives are comparable [9]. A generic linear ordering can be diagrammed as an acyclic line or

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chain of elements containing the alternatives of interest, linked by arrows depicting the strict preference relationship. In this conventional representation, the most preferred alternatives are positioned at the top. Two generic alternatives are always comparable, since there exist a path from the first to the second one (or vice versa) that is directed downwards.

The EYA is more versatile since admits orderings with omitted and/or incomparable alternatives, i.e., orderings that, according to the Mathematics' Order theory, are classified as *partial* [11]. This type of ordering can be diagrammed as a graph with branches, which determine different possible paths from the element(s) at the top to that one(s) at the bottom. If two alternatives are not comparable, there exists no direct path from the first to the second one (or *viceversa*).

The first step of this phase is to transform each (partial) ordering with incomparabilities into a set of linear sub-orderings. Precisely, a partial ordering can be artificially split into p linear sub-orderings, corresponding to the possible paths from the top to the bottom element(s). Obviously, the number of paths depends on the configuration of the relevant graph (e.g., amount and position of the branches). For the purpose of example, let us consider the preference orderings illustrated in Fig. 1, in which the agents' importance ordering is assumed to be $D_4 > (D_2 \sim D_3) > D_1$. It can be noticed that the (partial) ordering by agent D_1 includes $p = 2$ possible paths (A and B); therefore, this ordering is turned into two linear sub-orderings, D_{1A} and D_{1B}.

Each alternative in the sub-orderings is associated with a conventional number of occurrences, fractionalized with respect to the number of sub-orderings where the alternative is present. E.g., for c and b, the fractional number of occurrences is 1/2 as these alternatives are contained in both the sub-orderings D_{1A} and D_{1B}. The relative importance associated with each linear sub-ordering is that of the relevant source (partial) ordering.

Next, linear (sub-)orderings are turned into preference vectors, according to the following convention. We place the alternatives as they appear in the ordering, with the most preferred one(s) in the top positions. If at any point $t > 1$ alternatives are tied (i.e., indifferent), we place them in the same element and then place the null set ("Null") in the next $t - 1$ lower positions. Although there are six total alternatives (a, b, c, d, e and f), some of them may be omitted in a certain vector; therefore the number of elements (n_i) can change from a vector to one other. Table 1 exemplifies the construction of the preference vectors from the orderings in Fig. 1.

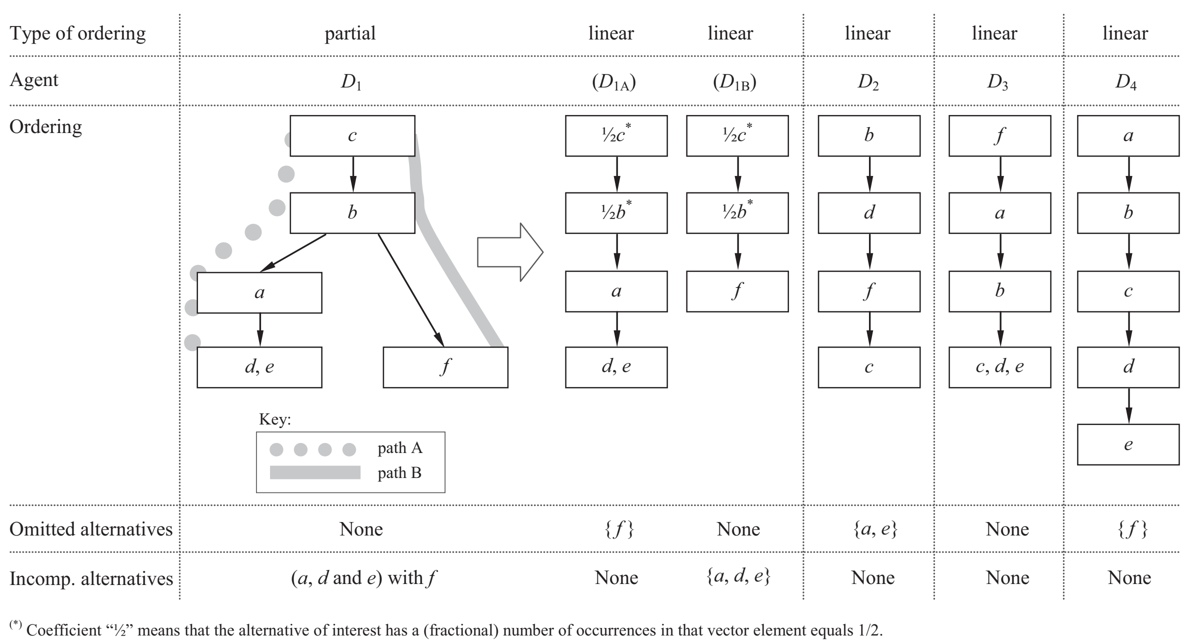

Fig. 1. Graphical representation of the preference orderings by four fictitious agents (D_1 to D_4). The alternatives of interest are a, b, c, d, e and f. The ordering by D_1 has two paths, therefore it is turned into two linear sub-orderings (D_{1A} and D_{1B}). The agents' importance ordering is assumed to be $D_4 > (D_2 \sim D_3) > D_1$.

Table 1
Construction of preference vectors for the linear (sub-)orderings in Fig. 1.

Agent	D_{1A}		D_{1B}		D_2		D_3		D_4	
Orderings	$c > b > a > (d \sim e)$		$c > b > f$		$b > d > f > c$		$f > a > b > (c \sim d \sim e)$		$a > b > c > d > e$	
No. of alternatives (n_i)	5		3		4		6		5	
Omitted alternative(s)	{f}		{a, d, e}		{a, e}		Null		{f}	
Preference vectors	$f_{1A,\,j}$	Elem.	$f_{1B,\,j}$	Elem.	$f_{2,\,j}$	Elem.	$f_{3,\,j}$	Elem.	$f_{4,\,j}$	Elem.
	1	{½c}	1.00	{½c}	1.00	{b}	1.00	{f}	1.00	{a}
	0.80	{½b}	0.67	{½b}	0.75	{d}	0.83	{a}	0.80	{b}
	0.60	{a}	0.33	{f}	0.50	{f}	0.67	{b}	0.60	{c}
	0.40	{d, e}			0.25	{c}	0.50	{c, d, e}	0.40	{d}
	0.20	Null					0.33	Null	0.20	{e}
							0.17	Null		

$f_{i,j} = j/n_i$ is the cumulative relative frequency referring to the j-th element of an i-th vector.