Measurement 88 (2016) 441–455

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Construction of orthogonal projector for the damage identification by measured substructural flexibility

Shun Weng [a,1], Hong-Ping Zhu [a,*], Peng-Hui Li [a], Yong Xia [b,2], Ling Ye [a]

[a] School of Civil Engineering & Mechanics, Huazhong University of Science and Technology, Wuhan, Hubei, PR China
[b] Department of Civil & Structural Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Available online 5 January 2016

Keywords:
Substructuring method
Damage identification
Rigid body modes
Stiffness matrix
Flexibility matrix
Orthogonal projector

ABSTRACT

The substructuring methods have been popularly used in model updating, system identification and damage assessment. In the substructuring methods, the global structure is divided into free–free substructures. The independent substructures move freely, and their stiffness matrix is singular and rank-deficient. The flexibility matrix of the free–free substructure, which is associated with the inverse of the stiffness matrix, is not easy to be determined. This study expands on the previous research of the substructuring methods by taking a deeper look at the analysis of a free–free substructure. An orthogonal projector is formulated to add/remove the rigid body components from the generalized stiffness and flexibility matrices of a free–free substructure, and thus make the substructural flexibility useful to model updating or damage identification. The orthogonal projector is derived both for the full and partial measured flexibility, and it can remove all rigid body components regardless its participation factor. The accuracy of the proposed method in extraction of the free–free flexibility and in damage identification is verified by an experimental beam. The properties addressed in this paper are not limited to be used for the analysis of a free–free substructure in many substructuring methods, and they are promising to be generalized to a range of analysis relevant to a free–free structure.

1. Introduction

In the past several decades, a large number of long-term structural health monitoring (SHM) systems have been designed and implemented worldwide on civil engineering structures such as large-scale bridges and high-rise buildings [1–5]. The accurate and efficient model updating and damage detection is significant for the long-term SHM systems. The dynamically measured flexibilities, residual flexibility and local flexibility are frequently used as the indexes for model updating and damage identification. It is observed that the flexibility is more sensitive to damage than the natural frequency or mode shape [6–8]. In particular, the local flexibility is inherently more sensitive to the local damage than the modal flexibility on the global structure, and the calculation of local flexibility attracts many research attention [9–11].

The substructuring methods have been extensively utilized in extraction of local flexibility or local stiffness of a structure [12–14]. The substructuring methods possess many advantages than the traditional global methods which analyze a structure as a whole. First, as the global structure is replaced by smaller and more manageable substructures, it is much easier and quicker to analyze the small system matrices. Second, the substructuring

* Corresponding author. Tel.: +86 27 8754 2631; fax: +86 27 8754 2221.
E-mail addresses: wengshun@mail.hust.edu.cn (S. Weng), hpzhu@hust.edu.cn (H.-P. Zhu), phlee@hust.edu.cn (P.-H. Li), y.xia@polyu.edu.hk (Y. Xia).
[1] Tel.: +86 27 8754 7208; fax: +86 27 8754 2221.
[2] Tel.: +852 2766 6066; fax: +852 2334 6389.

http://dx.doi.org/10.1016/j.measurement.2015.12.047
0263-2241/

methods allow for the analysis of local parts. When the substructuring method is applied in model updating or damage identification, only one or a few substructures are involved in an optimization procedure. The size of the model and the number of uncertain parameters are much smaller than those in the global structure. Finally, in practical testing, the experimental instruments can be saved if it is necessary to measure the whole structure only for one or more substructures [9–25].

Alvin and Park [15] proposed a force method to extract the substructural flexibility matrices. Doebling and Peterson [16] and Park and Felippa [17] disassembled the measured stiffness matrix and flexibility matrix into substructural stiffness matrices by projecting the measured matrices onto substructural strain energy distribution. Terrell et al. [18] proposed a substructure parameterisation technique, and the eigenvalues and eigenvectors of the super-elements were used for model updating and damage identification. Weng [12–14] proposed an inverse substructuring method to disassemble the modal properties of the global structure to the substructure level by satisfying the constraints at the interfaces. Afterwards, the independent substructures can be singled out to be used for the static analysis, dynamic analysis, nonlinear analysis, fatigue analysis and so forth.

The substructuring methods require dividing the global structure into independent free or fixed substructures. After partition, the substructures are usually analyzed independently under the free–free constraints. The substructural movement is usually contributed by both the rigid body motion and deformational motion. The rigid body motion of a free structure is always undetermined or even infinite. It is necessary to remove the rigid body components and thus reflect a real property of a structure. In addition, since a free–free structure includes the rigid body motion, its stiffness matrix **K** is singular and rank-deficient. The singular value decomposition of **K** is not only expensive, but notoriously sensitive to rank decisions when carried out in floating-point arithmetic [26,27]. In consequence, the flexibility matrix and residual flexibility matrices, associated with the inverse of the rank-deficient stiffness matrix, are not easy to be determined [27]. Some researchers avoided the rigid body modes by introducing a small shift on the rank deficient stiffness matrix [28] or extract the Moore–Penrose generalized inverses [29] of the stiffness matrix. This inevitably introduces some errors and computationally time consuming.

This paper addresses some frequently encountered difficulties associated with the analysis of the free substructures when the authors studied on the substructuring methods in the previous research [12–14,21–25]. In our previous work, the zero-frequency modes are solved by an shift eigensolver, and the zero frequency is replace by a small value, not zero exactly. Afterwards, the rigid body modes are treated equivalently with the deformational modes to be analyzed. This inevitably introduces some errors, although it is acceptable for most engineering application. In this paper, an orthogonal projector is proposed to remove the rigid body components of the generalized stiffness and flexibility matrices which are contributed by both the deformational components and the rigid body components. The proposed orthogonal projector is used to extract the substructural modal flexibility matrices that are disassembled from the global flexibility, and thus makes them applicable in the substructure-based model updating and damage identification. The orthogonal projector is derived for both the full measurement and partial measurement of flexibility. The formulae proposed in this paper are not only useful for the analysis of a free–free substructure in many substructuring methods, but also generally applicable in the analysis of a free structure. For example, the measured flexibility is frequently used for damage identification, and the measured flexibility is measured under the free condition [7,30]. The proposed orthogonal projector can be employed to extract the modal flexibility from the flexibility matrices measured under the free condition.

2. Construction of orthogonal projector

2.1. Eigenanalysis for a general singular matrix

If matrix **A** is a nondefective square matrix of order N, which may be unsymmetric and singular, matrix **A** is decoupled by its eigenvalues and eigenvectors as [26]

$$\mathbf{A} = \sum_i \lambda_i \{\phi_i\}\{\varphi_i\}^T \tag{1}$$

$$\phi_i^T \varphi_j = \delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker delta, λ_i are the eigenvalues, and ϕ_i and φ_i are the associated left and right bi-orthonormalized eigenvectors, respectively. The inverse or the Moore–Penrose inverse has the form of

$$\mathbf{A}^{-1} = \sum_i \frac{1}{\lambda_i}\{\phi_i\}\{\varphi_i\}^T \quad \text{or} \quad \mathbf{A}^{+} = \sum_i \frac{1}{\lambda_i}\{\phi_i\}\{\varphi_i\}^T \tag{3}$$

In structural engineering, a structure with N degrees of freedom (DOFs) has the stiffness matrix **K**, which is a symmetric and nondefective matrix. In physical viewpoint, the columns of the stiffness matrix **K** gives the loads to generate an unit displacement on a DOF. The stiffness matrix **K** of a linearly elastic structure relates node displacements to node forces through the stiffness equation [31]

$$\mathbf{K}\{x\} = f \tag{4}$$

where f includes the external force or constraints. According to Eq. (1), the stiffness matrix can be decoupled by the normalized eigenvectors as

$$\mathbf{K} = \sum_{i=1}^{N} \lambda_i (\phi_i)(\phi_i)^T = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^T \tag{5}$$

where $\mathbf{\Lambda} = Diag(\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_N)$ are the eigenvalues, and $\mathbf{\Phi} = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_N]$ are the corresponding orthogonal eigenvectors. They satisfy the following orthogonal relation

$$\mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} = \mathbf{\Lambda}, \quad \mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I} \tag{6}$$

A flexibility matrix has a very straightforward physical interpretation: the displacement response caused by an