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A conceptual framework for concept definition in measurement: The case of 'sensitivity' ☆,☆☆

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ABSTRACT

The concept 'sensitivity' has multiple and sometimes incompatible usages and definitions, as they can be found in the scientific and technical literature. A strategy is proposed toward a conceptual framework in which sensitivity is qualitatively intended as a feature of a black box behavior and quantitatively is defined according to specific evaluation types (interval/ratio, ordinal, nominal) for both deterministic and stochastic behaviors. The proposed formal definitions characterize stochastic sensitivity as constituted of "effective" and "confounding" components, that can be simultaneously present and contribute to a desirable and unwanted increment of global sensitivity respectively. Two examples taken from the context of imaging systems and image-based measuring systems, in which sensitivity is computed in presence of non-negligible uncertainty sources, provide some hints on the usefulness of the proposed framework.

1. Introduction

In all non-purely formal bodies of knowledge, and thus in experimental sciences and technology in particular, the relevant concepts are conveyed not only through mathematics but also by means of linguistic expressions, at least with the aim of interpreting mathematical constructs in terms of empirical entities. While maybe unavoidable, such usage of natural language is not exempt from problems, as it may lead to both synonyms (different terms designating the same concept) and polysemies (different concepts designated by the same term) [2]. Synonyms generate redundancy but their potentially negative effects can be easily prevented, for example by means of a thesaurus. On the other hand, polysemies are sources of ambiguities, whose presence reduces the chance or increases the cost of correct communication.

Of course, such issues are mainly due to traditions and consolidated usages of linguistic expressions and their relations to concepts, not to concepts as such. It is not amazing then that measurement, a process exploited in so many fields and since so long time, is particularly affected by an ambiguous lexicon. The plethora of specific measurement-related sublanguages hinders the inter-disciplinary communication and emphasizes an artificial separation between science(s) and society. Hence,

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contributing to bridge these gaps seems to be a promising, worthwhile effort.

It is the goal explicitly pursued by the Joint Committee for Guides in Metrology (JCGM) in the development of the *International vocabulary of metrology – Basic and general concepts and associated terms* (VIM) [3]. Building on the traditional ground of measurement of physical quantities, which was the basic scope of its first two editions, the current, third edition of the VIM explicitly aims at a broader reach, being "meant to be a common reference for scientists and engineers – including physicists, chemists, medical scientists – as well as for both teachers and practitioners involved in planning or performing measurements, irrespective of the level of measurement uncertainty and irrespective of the field of application. It is also meant to be a reference for governmental and inter-governmental bodies, trade associations, accreditation bodies, regulators, and professional societies." [3, Scope].

The VIM is then an important step toward a widespread, universally agreed concept system and lexicon for measurement science. On the other hand, the many existing sublanguages about measurement make the endeavor a complex and delicate one: whenever multiple concepts for the same term are found in the scientific and technical literature, a comparative study should be done to identify an appropriate, encompassing definition, or at least to emphasize the reasons and the nature of such multiplicity, including the possible differences in scope. An explicit strategy to produce consistent definitions/presentation should be adopted, particularly in the case the same concept is both qualitatively and quantitatively characterized and the two layers have to be properly coordinated.[1]

This paper is aimed at proposing a possible example of such a strategy in reference to the concept 'sensitivity', a case that at the same time is particularly difficult for its many, sometimes markedly different, characterizations and is relevant in many scientific and technical fields: the lessons learned might be then replicable to other measurement-related concepts. Hence this is in line with the attempt of the third edition of the VIM to widen the scientific and technical community to which the vocabulary is devoted.

2. Framing the concept 'sensitivity'

The concept 'sensitivity' is defined in so many contexts and with so many specifications that one might doubt that such a polysemy can ever be solved. A first problem is: sensitivity of what? Indeed, by means of a feature termed "sensitivity" different entities are characterized: measuring instruments and systems but also, e.g., methods, tests, and algorithms.[2]

Let us then adopt a black box (meta-)modeling strategy, by hypothesizing that sensitivity is a feature of an entity X under the only conditions that X is not static, i.e., it can produce different outputs $Y(t)$ depending on its input $U(t)$, and possibly a random contribution $N(t)$. The black box assumption implies that the analytical form of the input–output relation, generically denoted with f in the following, might not be known.[3] Furthermore, no constraints are given on the ranges and types of U and Y and on time domain, allowed to be either continuous or discrete.

On this basis some significant examples of the definition of 'sensitivity' follow, preliminarily classified in two general categories – let us denote them as α and β – in terms of this input–output characterization.

α. Sensitivity as $\Delta y/\Delta u$, where $u \in U$ and $y = f(u) \in Y$, possibly under specified conditions.

1. *Sensitivity of a measuring system*: "the quotient of the change in an indication of a measuring system and the corresponding change in a value of a quantity being measured"; VIM, 4.12 [3].
2. *Photocathode sensitivity*: "ratio of the photoelectric emission current from the photocathode to the incident luminous flux under specified conditions of illumination"; IEV, 394-38-11, Nuclear instrumentation – Instruments, systems, equipment and detectors/Characteristics of radiation detectors (all IEV definitions are taken from [5]).
3. *Sensitivity of a measuring assembly*: "for a given value of the measured quantity, ratio of the variation of the observed variable to the corresponding variation of the measured quantity"; IEV: 394-39-07, Nuclear instrumentation – Instruments, systems, equipment and detectors/ Characteristics of radiation measuring assemblies.
4. *Dynamic sensitivity*: "under stated conditions of operation, the quotient of the variation of the photoelectric current of the device by the initiating small variation of the incident radiant power or luminous flux"; IEV, 531-44-25, Electronic tubes/General properties and quantities of photosensitive tubes.

[1] The VIM3 offers some interesting examples of the qualitative vs quantitative distinction: "Measurement precision is usually expressed numerically by measures of imprecision..." [3, 2.15 N1]; "Measurement trueness is not a quantity and thus cannot be expressed numerically, but measures for closeness of agreement are given..." [3, 2.14 N1]; "The concept 'measurement accuracy' is not a quantity and is not given a numerical quantity value." [3, 2.13 N1]. It is the situation of three concepts with a qualitative definition and three different treatments as for their quantitative counterparts, without stated reasons justifying this difference.

[2] In this paper the distinction between terms referring to objects, or concepts, or terms is critical. Hence we adopt the notational convention from ISO standards, e.g., [4]. A term, and more generally a linguistic expression, referring to: – itself, i.e., a term, is *delimited by double quotes*; – a concept, i.e., its meaning, is *delimited by single quotes*; – an object, i.e., its referent, is *not delimited*. Hence, (the concept) 'sensitivity' is expressed in English by (the term) "sensitivity" and is about (the object) sensitivity. The lack of delimiters around terms for objects (i.e., entities of the world) follows an economic principle: in everyday writing, we usually intend to refer to objects, and not to concepts or terms.

[3] We will adopt the notational convention of denoting properties, typically modeled as random variables, by upper case characters and their values/occurrences by lower case characters. In general X is a multiple-input and multiple-output (MIMO) system, so that its input and output are vector properties. For the sake of simplicity, in the following we will assume that U and Y are scalars, then single components of these vectors.