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Non recursive multi-harmonic least squares fitting for grid frequency estimation

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ABSTRACT

A new method to approximate the least squares multi-harmonic fitting is proposed. The basic idea is to expand in Taylor's series the derivative on ω of the least squares cost function around a central value, so reducing the frequency estimation to a calculation of a root of a polynomial. In this way the method provides a frequency estimation in a closed form avoiding the recursion that is necessary in the classical approach. The results show that the proposed algorithm reaches the Cramer–Rao bound in a narrow range of frequency around a pre-estimation. Increasing the approximation orders of the Taylor's expansion the range of maximum accuracy widens. This method is particularly suitable in grid frequency estimation due its low variability. The proposed algorithm, preserving the accuracy, requires an execution time up to 8 times lower compared to a single iteration of the classical recursive approach.

1. Introduction

Frequency estimation is a challenge in many research field. In the Power System the accurate knowledge of the fundamental frequency and of its time variations is very important because it is an index of the balance between the production and the consumption of the energy in a wide interconnected area. Moreover, the actual frequency is useful to improve the performance of many algorithms both for the accurate estimation of other waveform parameters (i.e., the harmonic distortion) and for the detection of power quality events.

In the literature a great variety of approaches to estimate the frequency of a sampled periodic waveform have been proposed.

A number of methods are related to the frequency tracking problem which have the aim to correctly estimate the frequency under non-stationary conditions. These algorithms are usually characterized by a low computational burden and they are particularly suitable for the extraction of the frequency in real-time applications. The most popular approaches are based on demodulation techniques [1], on Phase Locked Loop [2], on Notch Adaptive Filtering [3], on Kalman filtering [4] and, recently, on hybrid adaptive system based on the Fast Fourier Transform [5]. In these papers the algorithms performances are evaluated mainly in terms of speed and quality of the frequency tracking.

On the other hand, in the literature several techniques have been proposed with the primary objective of maximizing the accuracy of the frequency estimation for a steady-state periodic signals in presence of additive noise and harmonic distortion. In these cases, the simulations are conducted considering the frequency constant in a time window. This work is focused on this latter condition.

In [6] a comparison of different algorithms shows as the interpolation of DFT spectrum in the neighborhood of the fundamental tone is a good compromise between accuracy and computational complexity.

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It can be found several approaches based on the method of least squares. Some algorithms exploit mathematical relations between consecutive samples and use a least squares approach to reduce the noise effect [7,8]. Recently Kusljevic [9] generalized the algorithm proposed in [7] and compared it with other methods confirming the advantages of the Park approach in terms of estimation accuracy in presence of noise and harmonic distortion.

Another approach based on least squares is the curve fitting. The standard IEEE-STD-1057 [10] proposes a method to extract the parameters of a periodic signal using a sinusoidal model. The method is well analyzed in [11] but it produces biased estimation in presence of harmonic pollution [12]. To reduce the influence of the harmonics on the estimation it is necessary to include them in the fitting model [13]. Some results using multi-harmonic fitting have been discussed in [14].

Frequency estimation in both sinusoidal and multi-harmonic fitting [11,14–16] is a non linear optimization problem and requires an iterative method to obtain the value. In addition, the multi-harmonic fitting has some other problems that make it not suitable for real-time applications, despite its theoretically high accuracy:

- The high number of parameters causes convergence problem and the pre-estimation of the harmonic amplitudes is necessary in order to improve the performance [15].
- The high computational demand especially for the computation of the pseudo-inverse matrix.

This work has the aim to propose a method to extract the frequency using a multi-harmonic fitting avoiding the recursion and overcoming the problems listed before. The proposed method is an extension of the one proposed in [17] that use a sinusoidal model. The new formulation allows to improve the performance in presence of harmonic distortion.

The basic idea consists in the approximation of the derivative of least squares cost function using the Taylor's series expansion around a pre-estimated value. In this way, the frequency value that maximizes the cost function can be expressed in closed form as a root of Taylor's polynomial. In the paper different approximation grade will be analyzed and compared.

In the first section the method will be derived starting from the classical approach. In the second section the performance of the algorithm obtained through simulations in different conditions of noise and harmonic pollution will be shown. The test signals used for the analysis reflect the typical features of the grid voltage. In the last section, a comparison of the performance in terms of estimation accuracy and execution time between the proposed method and other algorithms proposed in the literature will be shown.

The estimation error of the proposed method is very close to the Cramer–Rao lower bound in the typical range of variability of the grid frequency. At the same time, compared to a single iteration of the classical multi-harmonic fitting, the computational burden results up to 2 times lower using the third order approximation and up to 8 times lower using the second order approximation.

2. The proposed method

A multi-harmonic model consists of a sum of sinusoids with frequencies integer multiple of a fundamental value ω. Each harmonic has a proper amplitude A_h and phase φ_h:

$$\sum_{h=1}^{H} A_h \sin(h\omega t + \varphi_h) \tag{1}$$

Including the zero-order harmonic, it is possible to generalize the model considering an offset value. In this work, we consider a model with a null offset.

The curve fitting is a technique to estimate the parameters of a model that allows to best fit a data record $y[n]$. The most known method to find these parameters is the least squares rule: the searched parameters are the ones minimizing a cost function $J(\boldsymbol{\theta})$ defined as the sum of the squared difference between the data and its parametric model.

In the multi-harmonic fitting, the cost function can be written as:

$$J(\boldsymbol{\theta}) = \int_0^T \left(y(t) - \sum_{h=1}^{H} \alpha_h \sin(h\omega t) + \beta_h \cos(h\omega t) \right)^2 dt \tag{2}$$

In the expression (2) the observed data $y[n]$ is considered as a time-continuous signal $y(t)$. In this way, the classical sum of the squares becomes an integral defined on a time window T and it is possible to treat analytically the least squares problem. The vector $\boldsymbol{\theta} = [\omega, \alpha_1, \beta_1 \ \ldots \ \alpha_H, \beta_H]$ contains the $2H + 1$ parameters that must be estimated.

The estimations of α_h and β_h are a linear least squares problem while the estimation of the frequency ω is a non-linear problem.

The two problems can be separated [16], and the linear problem can be approached solving the following linear system, producing the expression of $\widehat{\alpha}_h$ e $\widehat{\beta}_h$ as function of ω:

$$\left[\widehat{\alpha}_1 \quad \widehat{\beta}_1 \quad \ldots \quad \widehat{\alpha}_H \quad \widehat{\beta}_H \right] = (\boldsymbol{D^T} \cdot \boldsymbol{D})^{-1} \cdot (\boldsymbol{D^T} \cdot y) \tag{3}$$

where

$$\boldsymbol{D} = [\sin(\omega t) \quad \cos(\omega t) \quad \ldots \quad \sin(H\omega t) \quad \cos(H\omega t)] \tag{4}$$

$\boldsymbol{D^T} \cdot \boldsymbol{D}$ is a squares matrix of dimension $2H \times 2H$ where each (i, j) element is the scalar product between the i-th and j-th element of $\boldsymbol{D}$. In continuous form:

$$\boldsymbol{D^T} \cdot \boldsymbol{D} = \begin{bmatrix} \int_0^T s_1^2 dt & \int_0^T s_1 c_1 dt & \ldots & \int_0^T s_1 s_H dt & \int_0^T s_1 c_H dt \\ \int_0^T c_1 s_1 dt & \int_0^T c_1^2 dt & \ldots & \int_0^T c_1 s_H dt & \int_0^T c_1 c_H dt \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \int_0^T s_H s_1 dt & \int_0^T s_H c_1 dt & \ldots & \int_0^T s_H^2 dt & \int_0^T s_H c_H dt \\ \int_0^T c_H s_1 dt & \int_0^T c_H c_1 dt & \ldots & \int_0^T c_H s_H dt & \int_0^T c_H^2 dt \end{bmatrix} \tag{5}$$

where