ARTICLE IN PRESS

Economic Modelling xxx (2017) 1–9

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Estimating concave substitution possibilities with non-stationary data using the dynamic linear logit demand model

Timothy J. Considine

University of Wyoming, Dept. 3985, 1000 E. University Ave., College of Business 323E, Laramie, WY 82071, USA

ARTICLE INFO

JEL Classifications:
C51
D24
Q31

Keywords:
Dynamic
Substitution
Concavity
Linear logit model

ABSTRACT

Measuring substitution possibilities is crucial for estimating the costs and benefits of climate, trade, banking, and many other policy issues. This paper addresses two problems encountered when modeling substitution: spurious correlations arising from data with trends and violations of the law of demand. This paper shows how the dynamic linear logit model addresses these two problems. First, the model allows adjustment in quantities and, thereby, avoids spurious correlations arising from data with significant trends. Secondly, the linear nature of the substitution elasticities facilitates parameter restrictions to ensure that substitution estimates are consistent with the law of demand and the mathematical conditions consistent with cost minimizing behavior by producers. These features are illustrated by estimating the dynamic linear logit model for energy demand in the U.S. industrial sector. The empirical results demonstrate that the dynamic linear logit model is well suited for data with common trends and for ensuring robust and intuitively appealing estimates of demand elasticities and associated substitution possibilities.

1. Introduction

Substitution possibilities between factor inputs play a prominent role in many policy debates. The oil price shocks during the 1970s and early 1980s led to a series of econometric studies focused on whether energy and capital are complements or substitutes (see, for example, Thompson and Taylor, 1995). Elasticities of substitution are also important in estimating the costs of marketable pollution permit trading systems, examined by Considine and Larson (2006), and in assessing international trade policy, evaluated by Hertel et al. (2007). Substitution parameters also play a crucial role in computable general equilibrium models used to estimate the economic impacts of policies reducing greenhouse gas emissions, (see Antimiani et al., 2015; Karney, 2016). Yet another example of the importance of measuring factor substitution is the paper by Spierdijk et al. (2017) that examines how banks adjust factor inputs in response to the financial crisis of 2008.

Researchers face a number of challenges in estimating substitution possibilities. This study examines two common problems facing many economic modeling studies of input factor demand and substitution. The first problem arises from common trends in economic time series that may generate spurious correlations. The second problem is positive own price elasticities of demand, which is a counter-intuitive violation of the law of demand. This paper shows how the dynamic linear logit demand system developed by Considine and Mount (1984) can be used to address these two problems.

The time series properties of data are often overlooked in studies of substitution possibilities using demand systems. The fact that most economic time series are non-stationary, characterized by strong trends, is well known. At least in theory, a demand system should act as a cointegrating model relating quantities or cost shares with relative prices so that the remaining residual variation is stationary white noise. Most applications of the popular translog system do not address the problems associated with unit roots in the data, often ignoring significant serial correlation in residuals. Lewbel and Ng (2005) show that aggregation of consumers with heterogeneous preferences in a slowly changing population provides an explanation of non-stationary aggregate demand errors. Their empirical analysis involves modeling aggregation and estimating the model in first-differences.

A similar approach within a production context involves the classic dynamic adjustment model extended by Treadway (1971) to multifactor demand systems. Slow adjustments in quasi-fixed factors also could lead to non-stationary residuals. Incorporating partial adjustment models in cost share systems derived from flexible functional forms, however, is problematic due to the adding-up constraints required by these forms. Considine and Mount (1984) explore a way around this problem by developing the dynamic linear logit (DLL) specification of a cost share system, showing

E-mail address: tconsidi@uwyo.edu.

https://doi.org/10.1016/j.econmod.2017.12.021
Received 9 June 2017; Received in revised form 19 December 2017; Accepted 31 December 2017
Available online xxxx

Please cite this article in press as: Considine, T.J., , Estimating concave substitution possibilities with non-stationary data using the dynamic linear logit demand model, Economic Modelling (2017), https://doi.org/10.1016/j.econmod.2017.12.021

how the logit form facilitates dynamic adjustments in quantities as opposed to cost shares because adding-up and non-negativity are automatically guaranteed. The empirical analysis below using non-stationary time series illustrates that the dynamic adjustments in quantities within the DLL model removes autocorrelation. First differencing of the data to obtain stationary residuals estimates is not required.

Several studies have demonstrated other advantages of the DLL model. Tyrrell and Mount (1982) demonstrate that the logit approach allows estimation of nonlinear Engel curves. Bewley and Young (1987) show that the coefficients in another form of the linear logit specification are Slutsky parameters. Chavas and Segerson (1986) argue that the linear logit specification does not place any restrictions on autoregressive processes of structural error terms. The studies by Considine (1989a,b) and Dumagan and Mount (1996) demonstrate the stable global curvature properties of the model arising from the share weighted linear functions for the elasticities of demand. The classic form of the model imposes symmetry at a point, often chosen to be the mean cost shares. Applications of this model include studies by Mahmoud (1992), Shui et al. (1993), and Arnberg and Bjørner (2007). Another version of the model developed by Considine (1990) imposes symmetry at the predicted cost shares and is used by Jones (1995, 1996), Urga and Walters (2003), Bronnlund and Lundgren (2004), and Steinbuks (2012) in a variety of empirical applications.

Many of these studies justify the use of the dynamic linear logit model because it provides correctly signed own price elasticities for a wide range of cost shares in contrast to flexible functional forms, such as the translog. Violations of concavity with the DLL model, however, can occur, as demonstrated by Considine (1989b). Some of these violations are due to the omission of relevant explanatory variables, such as environmental regulations or fuel use regulations in energy demand models described by Considine (1989b).

Nevertheless, there may be empirical situations where the researcher exhausts the possibilities for model re-specification. In these cases, one option is to impose concavity, such as Diewert and Wales (1987), Moschini (1998), and Serletis et al. (2010). These methods typically impose concavity at a normalizing point and then proceed to check whether concavity holds at each observation. Oftentimes, even after imposing concavity locally, violations occur at other observations. Diewert and Wales (1987) also propose the Generalized Barnett and McFadden functional forms that allow global concavity but at the cost of a significant increase in the number of parameters. While the studies by Considine (1989a,b, 1990) show that the linear logit model of cost shares can satisfy concavity for all sample points in some empirical cases, they do not impose concavity at either the mean or the predicted cost shares. The stable properties of the linear logit model illustrated by Considine (1989a) suggest that if the imposition of concavity is necessary, concavity may hold for all or most observations. Accordingly, this paper shows how to impose concavity on the DLL model of cost shares. Our approach uses the strategy developed by Moschini (1998) by re-parameterizing the model into substitution derivatives and then using the Cholesky factorization to impose concavity.

The next section presents the basic features of the DLL model of cost shares providing detail for a four input example to clearly describe how the constraints derived from economic theory are imposed on the model and the nature of the substitution terms. The third section explains how concavity is imposed on the model. An empirical example of industrial energy demand is used in section four to illustrate the cointegrating properties of the model and the impact of concavity constraints on the estimated elasticities. For this empirical example, once concavity is imposed – either at the mean or at the predicted cost shares – concavity holds for all observations in the sample. The paper concludes with a summary of the major features of the model.

2. The linear logit model of cost shares

Considine and Mount (1984) showed that the logistic function, which is widely used in econometrics to model discrete choice, can be used to model a system of cost shares that, like probabilities, must remain non-negative and sum to one. Unlike the flexible functional form approach, their approach does not involve deriving the input demand or cost share equations from a cost function, although Considine (1990) demonstrates a recursive procedure for estimating a cost function with a linear logit cost share system.

Consider a dynamic linear logit model of cost shares developed by Considine and Mount (1984):

$$\frac{P_{it}Q_{it}}{C_t} = w_{it} = \frac{e^{f_{it}}}{\sum_{j=1}^{N} e^{f_{jt}}} \forall i, \tag{1}$$

where P_{it} is the price for input i in period t, Q_{it} is the corresponding quantity, C_t is the expenditures on the i inputs, w_{it} is the cost share for the ith input, and f_{it} functions are defined as follows:

$$f_{it} = \alpha_i + \sum_{j=1}^{N} \beta_{ij} \ln P_{jt} + \lambda \ln Q_{it-1} + \varepsilon_{it} \tag{2}$$

where the αs and βs are parameters to be estimated, and the εs are stochastic error terms. Considine and Mount (1984) show that the lagged quantities in (2) are consistent with the flexible accelerator model of dynamic input demand developed by Treadway (1971). Total expenditures, C_t, and other explanatory variables, such as measures of non-price induced technological change could be added to the right-hand side of (2) but are omitted here to simplify the model.

Taking the logarithm of (1) results in the following expression:

$$\ln w_{it} = f_{it} - \ln \sum_{j=1}^{N} e^{f_{jt}} \forall i. \tag{3}$$

The partial derivatives of (3) with respect to prices provide the cost share elasticities:

$$H_{ikt} = \frac{\partial \ln w_{it}}{\partial \ln P_{kt}} = \beta_{ik} - \sum_{j=1}^{N} w_{jt}\beta_{jk} \forall i, k. \tag{4}$$

Considine and Mount (1984) show that the price elasticities of demand are functions of these cost share elasticities:

$$E_{ikt} = \frac{P_{kt}}{Q_{it}} \frac{\partial Q_{it}}{\partial P_{kt}} = H_{ikt} + w_{kt} - \delta_{ik}. \tag{5}$$

where δ_{ik} is the Kronecker product so that $\delta_{ik} = 1$ when $k = i$ and $\delta_{ik} = 0$ when $k \neq i$.

Zero-degree homogeneity in prices implies that the price elasticities of demand for any one input with respect to changes in its price and prices for the other inputs sum to zero, or $\sum_{k=1}^{N} E_{ik} = 0$, which implies $\sum_{j=1}^{N} (\beta_{ij} - S_{jt}) = 0$, where $S_{jt} = \sum_{k=1}^{N} w_{kt}\beta_{kj}$. This constraint is imposed with N parameter restrictions, $\sum_{j=1}^{N} \beta_{ij} = 0$ so that (2) can be transformed as follows:

$$f_{it} = \alpha_i + \sum_{j=1}^{N} \beta_{ij} \ln\left(\frac{P_{jt}}{P_{nt}}\right) + \lambda \ln Q_{it-1} + \varepsilon_{it}. \tag{6}$$

The demand functions with zero-degree homogeneity in prices imposed implies that producers do not have money illusion and, hence, respond to relative prices in making factor input choices.

Symmetry of price effects, which follows from the neoclassical economics assumption of a twice continuously differentiable expenditure function, means that the cross partial derivatives with respect to prices, defined here as the substitution terms, and the share weighted cross price elasticities of demand are equal: