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Easy bootstrap-like estimation of asymptotic variances☆

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HIGHLIGHTS

- The bootstrap is useful for estimating standard errors in economic applications.
- It can be computationally cumbersome in complex models.
- This paper proposes a simpler method that only requires estimation in 1 dimension.
- The method is illustrated with censored least absolute deviations estimation.
- A related method is proposed for two-step estimators.

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ABSTRACT

The bootstrap is a convenient tool for calculating standard errors of the parameter estimates of complicated econometric models. Unfortunately, the bootstrap can be very time-consuming. In a recent paper, Honoré and Hu (2017), we propose a "Poor (Wo)man's Bootstrap" based on one-dimensional estimators. In this paper, we propose a modified, simpler method and illustrate its potential for estimating asymptotic variances.

1. Introduction

Most standard estimators for cross-sectional econometric models have asymptotic distribution of the form

$$\sqrt{n}\left(\widehat{\theta}-\theta_0\right) \xrightarrow{d} N\left(0, H^{-1}VH^{-1}\right) \tag{1}$$

where θ_0 is the k-dimensional parameter of interest, H and V are symmetric, positive definite matrices to be estimated. It is usually possible to get explicit expressions for H and V, but estimating them can be computationally difficult in complicated models. The bootstrap[1] provides a simple method for estimating $H^{-1}VH^{-1}$ directly.

One practical problem with the bootstrap is that it requires re-estimating the model a large number of times. This can be a limitation for complicated models where it is time-consuming to calculate the objective function that defines the estimator, or for estimators that are based on sample moments that are discontinuous in the parameter.

In Honoré and Hu (2017), we introduced a version of the bootstrap which is based on calculating one-dimensional estimators using a fixed set of directions in $\mathbb{R}^k$ for each bootstrap replication. The covariance of these one-dimensional estimators is then used

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1 The bootstrap can also be used to provide asymptotic refinements that can lead to more reliable inference in finite samples. That is not the topic of this note.

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to back out estimators of H and V via *nonlinear* least squares. The benefit of this approach is that it is often much easier to calculate one-dimensional than k-dimensional estimators.

In this note, we introduce a modified approach which permits using one-dimensional estimators in different directions in each bootstrap replication, and which makes it possible to back out estimators to H and V via *linear* regression. In order to highlight the idea behind the approach, we will be deliberately vague about the underlying regularity conditions.

Section 2 describes our basic idea in the context of an extremum estimator, but as mentioned, the approach applies equally well to GMM estimators. In Section 3, we illustrate the potential usefulness of the approach by considering Powell's (1984) Censored Least Absolute Deviations Estimator. We choose this example because quantile regression estimators provide a classical example where the matrix H in (1) cannot be estimated by a simple sample analog. Section 4 demonstrates how the proposed approach can be used to estimate the variance of two step estimators. Two step estimators also provide a classical example where it is cumbersome to estimate the variance of an estimator. Section 5 concludes.

2. Our modified approach

To fix ideas, consider an extremum estimator of the form

$$\widehat{\theta} = \arg\min_t \frac{1}{n}\sum_{i=1}^n q\left(z_i; t\right) \tag{2}$$

where z_i is the data for observation number i, n is the sample size, and $\theta_0 = \arg\min_t E\left[q\left(z_i; t\right)\right]$ is the true parameter value. Under random sampling and weak technical assumptions, (1) holds with $V = V\left[q'\left(z_i; \theta_0\right)\right]$ and $H = E\left[q''\left(z_i; \theta_0\right)\right]$, where the differentiation is with respect to the parameter. See for example Amemiya (1985). The insight in Honoré and Hu (2017) is to consider (infeasible) one-dimensional estimators of the form

$$\widehat{a}\left(\delta\right) = \arg\min_a \frac{1}{n}\sum_{i=1}^n q\left(z_i; \theta_0 + a\delta\right),$$

where δ is a fixed k-dimensional vector and a is a scalar. The joint asymptotic distribution of m such estimators, $\widehat{a}\left(\delta_1\right), \ldots, \widehat{a}\left(\delta_m\right)$, is asymptotically normal with asymptotic variance

$$\Omega = \left(C'\left(I\otimes H\right)C\right)^{-1}\left(D'VD\right)\left(C'\left(I\otimes H\right)C\right)^{-1}, \tag{3}$$

where I is an $m\times m$ identity matrix,

$$\underset{(k\times m)}{D} = \begin{pmatrix} \delta_1 & \delta_2 & \cdots & \delta_m \end{pmatrix} \quad \text{and}$$

$$\underset{(km\times m)}{C} = \begin{pmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_m \end{pmatrix}.$$

Eq. (3) implies the relationship,

$$\left(C'\left(I\otimes H\right)C\right)\Omega\left(C'\left(I\otimes H\right)C\right) = \left(D'VD\right). \tag{4}$$

Honoré and Hu (2017) proved that for suitably chosen directions, $\delta_1, \ldots, \delta_m$, Eq. (4) identifies[2] V and H from Ω, and proposed estimating V and H by nonlinear least squares after estimating Ω with the bootstrap. Honoré and Hu (2017) also demonstrated that the same approach can be used for GMM estimators.

The argument leading to (1) is almost always based on the representation

$$\widehat{\theta} - \theta_0 \approx H^{-1}\frac{1}{n}\sum_{i=1}^n s_i \tag{5}$$

where $\approx$ means that the two sides differ by a magnitude which is asymptotically negligible relative to the right hand side, and s_i is a function of the data for individual i. For example, for the extremum estimator in (2), $s_i = q'\left(z_i; \theta_0\right)$ when q is smooth in the parameter. The same basic argument applies to the bootstrap (see Hahn (1996)). Specifically, consider a bootstrap sample $\left\{z_i^b\right\}$ of size[3] n, where the z_i^b's are drawn with replacement from the empirical distribution of $\{z_i\}$. Standard asymptotic theory implies that in each bootstrap replication, b, the estimator, $\widehat{\theta}_b = \arg\min_t \frac{1}{n}\sum_{i=1}^n q\left(z_i^b; t\right)$ has the linear representation

$$\widehat{\theta}_b - \widehat{\theta} \approx H^{-1}\frac{1}{n}\sum_{i=1}^n s_i^b \tag{6}$$

for the same H as in (5).

As in Honoré and Hu (2017), this paper considers (infeasible) estimators of the form

$$\widehat{a}\left(\delta\right) = \arg\min_a \frac{1}{n}\sum_{i=1}^n q\left(z_i; \theta_0 + a\delta\right),$$

where δ is a fixed k-dimensional vector. These estimator have the representation

$$\widehat{a}\left(\delta\right) \approx \left(\delta' H\delta\right)^{-1}\delta'\frac{1}{n}\sum_{i=1}^n s_i$$

and the corresponding (feasible) estimators in a bootstrap sample,

$$\widehat{a}_b\left(\delta\right) = \arg\min_a \frac{1}{n}\sum_{i=1}^n q\left(z_i^b; \widehat{\theta} + a\delta\right),$$

have the representation

$$\widehat{a}_b\left(\delta\right) \approx \left(\delta' H\delta\right)^{-1}\delta'\frac{1}{n}\sum_{i=1}^n s_i^b. \tag{7}$$

Note that we can write (7) as

$$\left(\delta' H\delta\right)\widehat{a}_b\left(\delta\right) \approx \delta' s^b$$

where $s^b = \frac{1}{n}\sum_{i=1}^n s_i^b$. Equivalently

$$\widehat{a}_b\left(\delta\right)\left(\delta' H\delta\right) - \delta' s^b \approx 0 \tag{8}$$

or

$$\sum_{j,\ell}\left(\widehat{a}_b\left(\delta\right)\delta_j\delta_\ell\right)h_{j\ell} - \sum_j \delta_j s_j^b \approx 0, \tag{9}$$

where s_j^b is the j'th element of s^b, δ_j is the j'th element of δ. Since $h_{j\ell} = h_{\ell j}$, Eq. (9) can be written as

$$\sum_j\left(\widehat{a}_b\left(\delta\right)\delta_j\delta_j\right)h_{jj} + \sum_{\ell<j}\left(2\widehat{a}_b\left(\delta\right)\delta_j\delta_\ell\right)h_{j\ell} - \sum_j \delta_j s_j^b \approx 0. \tag{10}$$

As in Honoré and Hu (2017), the same idea applies to GMM estimators.

It is useful to think of (10) as a linear regression model where the parameters are the $h_{j\ell}$'s and the s_j^b's , the dependent variable is always 0 and (asymptotically) there is no error. Of course, for this to be useful, one needs to impose a scale normalization such

2 Except for an innocuous scale normalization.

3 In principle, the bootstrap sample size can differ from the actual sample size. We ignore this in order to keep the notation simpler.