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A generalization of Ramsey rule on discount rate with regime switching☆

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HIGHLIGHTS

- The Ramsey rule on the discount rate is extended to regime-dependent interest-rate formulas.
- The effect of the regime switching is dominant.
- Dynamic programming is used to derive the generalized equilibrium discount rate in closed form.

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ABSTRACT

I generalize the following rule of Ramsey (1928) on the discount rate with regime switching: the discount rate is the sum of the rate of pure time preference and the product of the consumption elasticity of marginal utility and the consumption growth rate. The Ramsey rule can be extended to regime-dependent interest-rate formulas for discounting future regime changes. Notwithstanding debate about empirically plausible values of the rate of pure time preference, I theoretically show that the effect of pure time preference is overwhelmingly dominated by the effect of the regime switching parameter. This is closely associated with consumption smoothing consequences across regimes.

1. Introduction

Since the seminal paper of Ramsey (1928), economists have assumed that agents are impatient. This can be modeled as agents who obtain their future utility at a discount from current utility. This has been applied to neoclassical economists' expected utility theory. The bottom line is that agents' lifetime utility is comprised of the sum of discounted flows of current and future utilities, thereby the discount is captured by the rate of pure time preference. However, it has been a challenge to get economists to shake hands on an appropriately agreed rate of pure time preference. In line with this, Ramsey (1928) claims

> One point should perhaps be emphasized more particularly; it is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination.

This is precisely the direction I would like to explore here. I generalize the following rule of Ramsey (1928) on the discount rate with regime switching:[1] the discount rate is the sum of the

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[1] In the aftermath of the global economic crisis of 2008, regime switching has attracted much interest recently to appropriately account for business cycle expansions and recessions. As Cochrane (2017) arguably states in the introduction of his paper,

> Asset prices and returns are correlated with business cycles. Stocks rise in good times, and fall in bad times. Real and nominal interest rates rise and fall with the business cycle. Stock returns and bond yield also help to forecast macroeconomic events such as GDP growth and inflation.

In light of the stylized behavior of economic cycles in the long run, regime switching models have become standard elements in economic modeling. Theoretical and empirical advances, depending on and extending Hamilton's (1989)

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rate of pure time preference and the product of the consumption elasticity of marginal utility and the consumption growth rate.[2] For analytical crispness, I assume that there are two regimes ("Bull" and "bear") with different fundamental parameters such as the expected rate and volatility of consumption growth. The Ramsey rule can be extended to regime-dependent interest-rate formulas for discounting future regime changes. The generalized-Ramsey formula for the discount rate with regime switching shows that the effect of pure time preference is overwhelmingly dominated by the effect of the regime switching parameter. This is closely associated with consumption smoothing consequences across regimes. Unlike the case without regime switching, agents are willing to achieve cross-regime consumption smoothing, which is reflected in the generalized-Ramsey formula.

2. The model

I consider an infinite-horizon economy with a single consumption good (the numeraire). Uncertainty is driven by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ in which a multi-dimensional Brownian motion and one Poisson process representing regime-switching risk are defined. All stochastic processes are adapted to $\{\mathcal{F}_t\}$, which is the P-augmentation of the filtration generated by the Brownian motion and Poisson process. All stated stochastic processes are assumed to be well defined, without explicitly stating the regularity conditions ensuring this.

Investment opportunities are represented by the financial market comprised of one instantaneously riskless bond in zero net supply, and multiple stocks each in constant net supply of 1 and paying dividends. The fundamental parameters in the investment opportunities are regime dependent. There are two regimes, "(B)ull" (regime B) and "(b)ear" (regime b). Regime i switches into regime j at the first time of an independent Poisson process with intensity δ_i, for $i, j \in \{B, b\}$. In regime i, the bond price B and the stock prices S are given by

$$dB(t) = r_i B(t)dt$$

and

$$dS(t) + D(t)dt = S(t)\{\mu_i dt + \sigma_i^\top dZ(t)\},$$

where r_i is the constant risk-free interest rate, $D(t)$ is the dividend vector process, μ_i is the constant mean vector, σ_i is the constant nonsingular standard deviation matrix, and $Z(t)$ is the Brownian motion with dimensionality equal to the number of stocks.

For the given adapted nonnegative consumption process $c(t)$ and adapted portfolio process $\pi(t)$, an agent accumulates her wealth $W(t)$ according to the following dynamic budget constraint: in regime i,

$$\begin{aligned} dW(t) &= \{r_i W(t) - c(t) + \pi(t)^\top(\mu_i - r_i \mathbf{1})\}dt + \pi(t)^\top \sigma_i^\top dZ(t), \\ W(0) &= w \geq 0, \end{aligned} \tag{1}$$

which is subject to the solvency constraint $W(t) \geq 0$, where $\mathbf{1}$ is a vector of 1's with dimensionality equal to the number of stocks. I denote by $\mathcal{A}(w)$ the set of admissible policies of consumption process $c(t)$ and portfolio process $\pi(t)$ such that the dynamic budget constraint stated above is satisfied.

Each representative agent is assumed to derive utility from consumption in the form of

$$\int_0^\infty e^{-\beta t} U\big(c(t)\big)dt,$$

where $\beta > 0$ is the rate of pure time preference (or equivalently, the rate of impatience), and $U(\cdot)$ measures the agent's utility and is twice continuously differentiable, strictly increasing, and strictly concave. Motivated by neoclassical economists' expected utility theory, the representative agent's optimization problem is described by finding the value function as follows: in regime i,

$$V_i(w) \equiv \sup_{(c,\pi)\in\mathcal{A}(w)} E\Big[\int_0^\infty e^{-\beta t} U\big(c(t)\big)dt\Big],$$

subject to regime switching with different fundamental parameters in the investment opportunities such as the expected rate and volatility of stock returns. The agent aims to maximize her expected utility from consumption by optimally controlling per-period consumption process $c(t)$ and portfolio process $\pi(t)$ that are regime dependent. After integrating out the Poisson process representing the regime-switching risk, the agent solves

$$V_i(w) = \sup_{(c,\pi)\in\mathcal{A}(w)} E\Big[\int_0^\infty e^{-(\beta+\delta_i)t}\Big\{U\big(c(t)\big) + \delta_i V_j\big(W(t)\big)\Big\}dt\Big]. \tag{2}$$

3. A generalized-Ramsey formula for discount rate

I examine a simple exchange economy in the style of Lucas (1978). The extension is that a representative agent faces regime-shift risk. The agent receives an endowment to be consumed in equilibrium and is assumed to trade a riskless asset and multiple risky assets entitling the owner to the dividend in the economy. The returns to these assets adjust to represent a no-trade equilibrium. The primitives of the equilibrium model are as follows. I simply assume that in regime $i \in \{B, b\}$, the exogenously given aggregate consumption process follows a geometric Brownian motion and it is given by

$$dD(t) = D(t)\{\mu_i^D dt + (\sigma_i^D)^\top dZ(t)\}, \tag{3}$$

where the expected instantaneous growth rate μ_i^D is the constant mean and the instantaneous volatility of growth rate σ_i^D is the constant standard deviation vector that may be regime dependent.

Definition 3.1. An equilibrium is a collection of (r_i, μ_i, σ_i) and optimal policies $\big(c(t), \pi(t)\big)$ such that the consumption good, stock, and bond markets clear, in other words,

$$\begin{aligned} c(t) &= D(t), \\ \pi^j(t) &= S^j(t), \quad j = 1, \dots, N, \\ W(t) &= \sum_{j=1}^N S^j(t), \end{aligned}$$

where N is the number of multiple stocks.

Now I derive a generalized-Ramsey formula for the discount rate under the widely adopted constant relative risk aversion (CRRA) utility function:

$$U\big(c(t)\big) = \frac{c(t)^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$ is the constant coefficient of relative risk aversion.

Theorem 3.1. *Under the CRRA utility function, the Ramsey rule on the discount rate can be generalized by the following formula: for $i, j \in \{B, b\}$, $i \neq j$, regime-dependent discount rates are given by*

$$r_i = \beta + \gamma\mu_i^D - \frac{1}{2}\gamma(1+\gamma)\|\sigma_i^D\|^2 + \delta_i\Big\{1 - \frac{M_j(r_j)}{M_i(r_i)}\Big\}, \tag{4}$$

where $M_i(r_i)$ and $M_j(r_j)$ satisfy the following system of algebraic equations:

$$-(\eta_i(r_i) + \delta_i)M_i(r_i) + \gamma M_i(r_i)^{1-1/\gamma} + \delta_i M_j(r_j) = 0,$$

seminal application of regime switching to economic recessions and expansions, have easily permitted the identification of stylized behavior of data such as time-varying properties.

[2] This well-known Ramsey rule on the discount rate has served as a key role in an optimal intertemporal allocation when dealing with the productivity of capital or fundamentally, the return on investment.