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Incentives in lottery contests with draws[☆]

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HIGHLIGHTS

- A private value lottery contest model with draws is considered.
- Draws increase (decrease) the strong (weak) contestant's effort incentive.
- Total effort is reduced after the introduction of a draw.
- Expected winner's effort can be higher if types are sufficiently dispersed.

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ABSTRACT

We investigate the incentive consequences of introducing the possibility of draws into lottery contests. Equilibrium total effort unambiguously decreases when draws are introduced, whereas the equilibrium expected winner's effort increases when the contestants' valuations of the prize become sufficiently dispersed.

1. Introduction

Contests that permit draws (or interchangeably, ties, gaps), in which no contestant wins the prize outright, are commonly observed in practice and have been extensively studied in the contest literature.[1] Intuitively, adding the possibility of a draw in a contest or a tournament softens competition between contestants and hence dampens their incentive to exert effort. This intuition is formalized and confirmed by Nti (1997) using a symmetric multi-player Tullock contest.

Even though draws reduce player incentive, previous studies have pointed out the potential merits of introducing draws into a contest/tournament. For instance, Nalebuff and Stiglitz (1983) and Eden (2007) show that although the introduction of draws weakens incentive, it can be optimal to the designer because each actual draw occurrence saves a payment to a contestant and increases the designer's expected profits. Recently, Imhof and Kräkel (2014) show that the designer strictly benefits from a gap if contestants are risk-averse because it provides partial insurance to the contestants on their income distributions and helps reduce the agency cost.

To the best of our knowledge, the extant literature has assumed that players are identical and has restricted its attention to the symmetric equilibria. In this paper, we relax the symmetry assumption and investigate the impact of the contestants' heterogeneity on their effort incentives. Specifically, we build a model based on Nti (1997) by allowing for heterogeneity among contestant winning values, in which no contract is provided and all agents

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[1] For a list that is indicative, but by no means exhaustive, see Nalebuff and Stiglitz (1983), Lazear and Rosen (1981), Eden (2007), Imhof and Kräkel (2014, 2016) for gaps in rank-order tournaments; see Nti (1997), Blavatskyy (2010), Jia (2012), Vesperoni and Yildizparlak (2017) for draws in imperfectly discriminating contests; see Gelder et al. (2015, 2016) for ties in perfectly discriminating contests (or equivalently, all-pay auctions).

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are risk-neutral. This setup allows us to abstract from the aforementioned wage-saving and insurance-provision motives from the designer, and to focus on the impact of draws on incentives.

We find that draws can benefit the designer from a pure incentive perspective. When there are two heterogeneous contestants, introducing a draw increases the strong player's effort incentives and decreases the weak player's effort level (Lemma 1). On the whole, total effort decreases with the presence of draws; however, if the players' types are sufficiently dispersed, the equilibrium expected winner's effort increases (Proposition 1). This result can be generalized to contests with more than two players (Proposition 2).

2. Model

There are $n \geq 2$ risk-neutral contestants competing for a prize. The value of the prize to contestant $i \in \{1, \ldots, n\}$ is $v_i > 0$, which is common knowledge. Without loss of generality, we order the contestants according to their valuations of the prize so that $v_1 \geq v_2 \geq \cdots \geq v_n > 0$. To win the prize, contestants exert irreversible effort simultaneously. Following Nti (1997), we assume that the winning probability of contestant i under effort profile $(x_1, \ldots, x_n)$ is given by

$$p_i(x_1, \ldots, x_n) = \begin{cases} \dfrac{x_i}{\sum_{i=1}^n x_i + s} & \text{if } \sum\limits_{i=1}^n x_i + s > 0 \\ \dfrac{1}{n} & \text{otherwise,} \end{cases}$$

where $s \geq 0$. Blavatskyy (2010) recently axiomatized this functional form.[2] The parameter s allows us to accommodate the possibility of a draw in a simple manner.[3] When $s = 0$, the winning probabilities across all contestants add up to one (i.e., $\sum_{i=1}^n p_i = 1$) and hence the prize is allocated to one of the contestants with certainty. When $s > 0$, a draw occurs with positive probability (i.e., $\frac{s}{\sum_{i=1}^n x_i + s} > 0$) and no contestant wins the prize.[4]

We assume that the contest designer's objective is to either (i) maximize the total effort of all contestants or (ii) maximize the expected winner's effort. The first objective function is commonly assumed in the contest literature. The second objective function is relevant in many contexts (e.g., research competitions) where the contest designer may care most about the performance of the winning contestant because only the winner's project will be executed (Baye and Hoppe, 2003; Serena, 2017).[5]

It is useful to introduce some notations before we proceed. Fixing s, denote contestant i's equilibrium effort by $x_i^*(s)$ for $i \in \{1, \ldots, n\}$. Similarly, denote the equilibrium total effort and expected winner's effort by $TE(s)$ and $WE(s)$ respectively. By definition, we have that

$$TE(s) \equiv \sum_{i=1}^n x_i^*(s), \tag{1}$$

and

$$WE(s) \equiv \sum_{i=1}^n p_i(x_1^*, \ldots, x_n^*) \cdot x_i^*(s) = \frac{\sum_{i=1}^n \left[x_i^*(s)\right]^2}{\sum_{i=1}^n x_i^*(s) + s}. \tag{2}$$

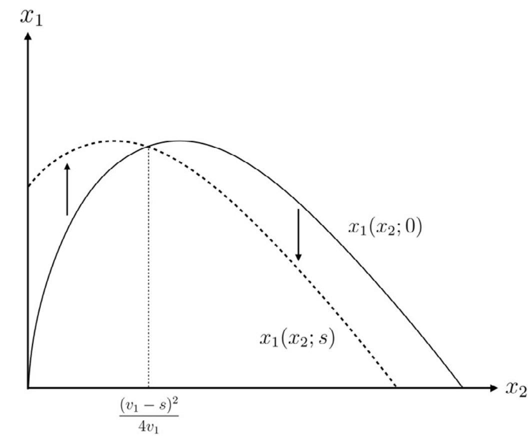

Fig. 1. Contestant 1's best response function.

3. Contest with $n = 2$ players

To explain the intuitions most cleanly, let us first consider a contest with two contestants. Fixing $s \geq 0$ and contestant 2's effort level x_2, contestant 1's response function, denoted by $x_1(x_2; s)$, can be derived as

$$x_1(x_2; s) = \max\left\{ \underbrace{\sqrt{v_1 \cdot (x_2 + s)}}_{\text{indirect effort boost effect}} - \underbrace{(x_2 + s)}_{\text{direct effort loss effect}}, 0 \right\}. \tag{3}$$

Contestant 2's best response function $x_2(x_1; s)$ can be derived similarly. From Eq. (3), introducing the possibility of a draw has two opposing effects. First, as the previous literature has pointed out, it *directly* weakens a contestant's effort incentive due to the decreased probability of winning for any effort level. Second, because the probability of a draw (i.e., $\frac{s}{x_1+x_2+s}$) depends on a contestant's effort, it also *indirectly* provides an incentive to a contestant to increase his effort level in order to avoid the loss suffered from a draw. Moreover, the latter indirect effect vanishes and will be dominated by the former direct effect as the rival's effort increases. Simple algebra shows that these two opposing effects cancel out at $x_2 = \frac{(v_1-s)^2}{4v_1}$. Therefore, contestant 1's effort will increase upon the introduction of a draw when the rival's effort level is low (i.e., $x_2 < \frac{(v_1-s)^2}{4v_1}$) and will decrease otherwise, as shown in Fig. 1.

The next lemma characterizes the equilibrium effort portfolio when s is sufficiently small.

Lemma 1. *Suppose $n = 2$ and $s < \frac{v_2^2}{v_1}$. Then the equilibrium portfolio $(x_1^*(s), x_2^*(s))$ is given by*

$$x_i^*(s) = y^*(s) - \frac{[y^*(s)]^2}{v_i} > 0, \text{ for } i = 1, 2, \tag{4}$$

where

$$y^*(s) = \frac{1 + \sqrt{1 + 4s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}}{2\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}. \tag{5}$$

Moreover, (i) if $v_1 = v_2$, then $x_i^*(s)$ is strictly decreasing in $s \in [0, v_2]$ for $i = 1, 2$; (ii) if $v_1 > v_2$, then $x_1^*(s)$ is strictly increasing in

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[2] See also Jia (2012); Jia et al. (2013) for a stochastic derivation.

[3] The parameter s is treated as a discount rate in a patent race context in Nti (1997).

[4] Alternatively, we can assume that each contestant receives an identical fraction of the prize in the case of a draw. Please see Footnote 6 for more discussions.

[5] The objective of maximizing the expected winner's effort in the imperfectly discriminating contest is parallel to the widely adopted objective of maximizing the expected highest effort in the perfectly discriminating contest (i.e., an all-pay auction).