Economics Letters 162 (2018) 49–52

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Weak convergence of local quantile treatment effect processes

Ju Hyun Kim [a], Byoung G. Park [b,*]

[a] Department of Economics, University of North Carolina at Chapel Hill, USA
[b] Department of Economics, University at Albany, 1400 Washington Avenue, Albany, NY 12222, USA

HIGHLIGHTS

- The weak convergence of the local quantile treatment effect (LQTE) estimator is established.
- The empirical bootstrap is proposed to consistently estimate the limiting distribution of the LQTE process.
- Examples are given to illustrate the use of the limiting distribution of the LQTE process.

ARTICLE INFO

Article history:
Received 11 August 2017
Received in revised form 20 October 2017
Accepted 23 October 2017
Available online 4 November 2017

JEL classification:
C13
C14
C15
C35
C36

Keywords:
Treatment effects
Quantile regression
Endogeneity
Semiparametric model
Weak convergence
Bootstrap

ABSTRACT

This paper considers a quantile regression *process* in the instrument variable model of Abadie et al. (2002). We extend pointwise analysis of local quantile treatment effects (LQTE) to the quantile process by establishing its weak convergence. We discuss the usefulness of our result in the context of hypothesis testing for the LQTE process.

1. Introduction

Since the seminal work of Imbens and Angrist (1994), the local treatment effects framework has drawn a considerable amount of attention in economics and econometrics. This approach has opened the way to allow for self-selection and unobserved heterogeneity in the inference for causal impacts. Abadie et al. (2002) developed an easy-to-compute method of estimating the local quantile treatment effects (LQTE) in semiparametric models. They focus on the *pointwise* inference at a given quantile level, but it has remained unanswered in the literature how to infer the quantile *process* over multiple quantile levels in the same setup. In this paper, we extend pointwise analysis to the quantile process by establishing its weak convergence, and propose the inference methods for the process.

The information on quantile processes provides us with key answers to interesting questions about the distributional impacts of some policy. For example, researchers may be interested in if people at all quantile levels benefit from some policy. This can be shown by stochastic dominance test between marginal distributions of potential outcomes. Also, they may ask how the benefits of some policy are distributed across different quantile levels, which can be tested by jointly comparing the impacts at different quantile levels.

There has been a large literature on the uniform inference for quantile regression processes since pioneering papers including Gutenbrunner and Jurecková (1992) and Koenker and Xiao (2002). For the quantile treatment effects, Firpo (2007), Firpo and Pinto (2016) and Ferreira et al. (2017) develop and apply estimators of unconditional distributional effects under the selection on

* Corresponding author.
E-mail addresses: juhkim@email.unc.edu (J.H. Kim), bpark2@albany.edu (B.G. Park).

https://doi.org/10.1016/j.econlet.2017.10.021

observables assumption. Chernozhukov and Hansen (2004, 2007) consider the uniform inference on conditional quantile treatment effects under selection on unobservables. While our work is closely related to the literature, it is the first paper developing uniform inference theory for the LQTE process in the model of Abadie et al. (2002).

The remaining part of the paper is organized as follows. In Section 2, we discuss the LQTE setup. Section 3 proposes estimation and inference methods by establishing weak convergence of local quantile treatment effect processes. Section 4 reports Monte Carlo simulation results to investigate the finite sample performance of our proposed method for hypothesis testing.

2. Setup of the local quantile treatment effect model

We consider the potential outcome framework with a binary treatment and a binary instrument. Let $D \in \{0, 1\}$ be the indicator for the treatment intake, and $Y_d \in \mathbb{R}$ be the potential outcome associated with the treatment state $d \in \{0, 1\}$. The observed outcome Y can be written as $Y = DY_1 + (1 - D)Y_0$. Suppose that we observe a binary instrument $Z \in \{0, 1\}$. The instrument affects the choice of treatment D. Let $D_z \in \{0, 1\}$ denote the potential treatment state when $Z = z$ with $z \in \{0, 1\}$. X is a vector of observed characteristics. In sum, the underlying model consists of variables $(Y_1, Y_0, D_1, D_0, X, Z)$, and the econometrician only observes a random sample $\{(Y_i, D_i, X_i, Z_i)\}_{i=1}^n$.

We impose the same assumptions as those in Abadie et al. (2002).

Assumption 1. The following assumptions hold conditional on X almost surely.

(i) Independence: (Y_1, Y_0, D_1, D_0) is jointly independent of Z.
(ii) Nontrivial Assignment: $\Pr(Z = 1|X) \in (0, 1)$.
(iii) First-Stage: $E[D_1|X] \neq E[D_0|X]$.
(iv) Monotonicity: $\Pr(D_1 \geq D_0|X) = 1$.

Assumption 1 is a restatement of the assumptions in Abadie et al. (2002) for our model. Assumption 1(i) is a standard independence assumption in the heterogeneous treatment effect model. Assumption 1(ii) is unlikely controversial for the discrete instrument in the literature. Assumption 1(iii) is the relevance condition for the instrument. Assumption 1(iv) is the monotonicity assumption, which is also known as the "no defier" assumption. It is the key identifying assumption.

The individuals with $D_1 > D_0$ are referred to as the compliers. Let $Q_{Y_d}(\tau|X, D_1 > D_0)$ denote the conditional τth quantile of Y_d given X and $D_1 > D_0$ for $d \in \{0, 1\}$ and $\tau \in (0, 1)$. It is well known that the conditional marginal distributions of the potential outcomes Y_0 and Y_1 given X are identified for compliers, which the following lemma in Abadie et al. (2002) formally states.

Lemma 1. *Under Assumption 1, $Q_{Y_d}(\tau|X, D_1 > D_0)$ is identified for $d \in \{0, 1\}$, $\tau \in (0, 1)$, and almost any X.*

Here we consider a known functional form as a semiparametric restriction, which is widely imposed in practice.

Assumption 2. Let $\mathcal{T}$ be a subinterval of $(0, 1)$. For any $\tau \in \mathcal{T}$, there exists $\theta_0(\tau) \in \Theta$ such that

$$Q_Y(\tau|D, X, D_1 > D_0) = h(D, X, \theta_0(\tau), \tau),$$

where $h : \{0, 1\} \times support(X) \times \Theta \times \mathcal{T} \to \mathbb{R}$ is a known function up to θ_0.

One particular case of the functional form of h is a linear conditional quantile model

$$h(D, X, \theta_0(\tau), \tau) = \alpha_0(\tau) \cdot D + X^\top \beta_0(\tau). \tag{1}$$

Here, $\theta_0(\tau) = (\alpha_0(\tau), \beta_0^\top(\tau))^\top$, and $\alpha_0(\tau)$ is the parameter representing the causal effect of D on the outcome in the model. Some interesting questions in policy evaluation involve hypotheses testing based on the quantile process $\alpha_0(\cdot)$ over the entire quantile.

Example 1. A policy maker might be interested in testing whether the treatment is beneficial to everyone. This test involves the following hypotheses:

$$H_0 : \alpha_0(\tau) \geq 0 \text{ for all } \tau \in \mathcal{T},$$
$$H_1 : \alpha_0(\tau) < 0 \text{ for some } \tau \in \mathcal{T}.$$

Example 2. The second example is the hypothesis testing for the constant quantile treatment effect. The constant effect across quantiles means that the treatment affects only the location of outcome, but not any other moments, in which $\alpha_0(\tau)$ is constant across all $\tau \in \mathcal{T}$. Then the hypotheses are formulated as follows:

$$H_0 : \alpha_0(\tau) \text{ is constant for all } \tau \in \mathcal{T},$$
$$H_1 : \alpha_0(\tau) \text{ varies with } \tau \in \mathcal{T}.$$

3. Estimation and inference

According to Theorem 3.3 in Koenker and Bassett (1978), the true parameter value θ_0 satisfies

$$E\left[\psi(Y, D, X, \theta_0, \tau)|D_1 > D_0\right] = 0,$$

where ψ is the gradient of the check function given by

$$\psi(Y, D, X, \theta, \tau) = \frac{\partial h(D, X, \theta(\tau), \tau)}{\partial \theta(\tau)} \cdot \left(\tau - 1\{Y \leq h(D, X, \theta(\tau), \tau)\}\right).$$

Note that this problem cannot be solved directly because the group of compliers is not identified. To convert this into a problem involving observed quantities only, we use a weighting function proposed in Abadie (2003). Let

$$\kappa_0^*(D, X, Z) = 1 - \frac{D \cdot (1 - Z)}{1 - \pi_0(X)} - \frac{(1 - D) \cdot Z}{\pi_0(X)},$$

where $\pi_0(X) = \Pr(Z = 1|X)$. The following lemma is given in Abadie (2003).

Lemma 2. *Let $g(Y, D, X)$ be any real function of (Y, D, X). Suppose that Assumption 1 holds and that $E\,|g(Y, D, X)| < \infty$. Then, we have*

$$E\left[g(Y, D, X)|D_1 > D_0\right] = \frac{1}{\Pr(D_1 > D_0)} \cdot E\left[\kappa_0^*(D, X, Z) \cdot g(Y, D, X)\right].$$

Now we can rewrite the parameter θ_0 as a solution to the following problem:

$$E\left[\kappa_0^*(D, X, Z) \cdot \psi(Y, D, X, \theta_0, \tau)\right] = 0 \tag{2}$$

for any $\tau \in \mathcal{T}$. However, the numerical algorithm does not ensure the global optimum because the weighting function κ^* turns negative when $D \neq Z$, which poses a nonconvex optimization problem. To address this issue, Abadie et al. (2002) proposed a modified version of the weighting function:

$$\kappa_0(U) = E[\kappa_0^*(D, X, Z)|U] = 1 - \frac{D \cdot (1 - \nu_0(U))}{1 - \pi_0(X)} - \frac{(1 - D) \cdot \nu_0(U)}{\pi_0(X)},$$