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Idiosyncratic volatility, returns, and mispricing: No real anomaly in sight

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ABSTRACT

Recent empirical evidence has shown that the relationship between idiosyncratic volatility and a stock's expected return depends on the pricing of the stock: it is negative among overvalued stocks and positive among undervalued ones. We provide both theoretical and numerical evidence that this risk-return relationship might be driven purely by mathematical properties of return distributions. Using a simulation-based approach, we document that even in completely random samples, the correlation between idiosyncratic risk and mean returns depends on the ex-post estimation of abnormal returns.

1. Introduction

The idiosyncratic volatility puzzle appears when stocks with low idiosyncratic risk outperform stocks with high idiosyncratic risk. Originally discovered by Ang et al. (2006), the phenomenon has subsequently been documented in numerous equity markets (e.g., Ang et al., 2009; Guo and Savickas, 2008).

Interestingly, Stambaugh et al. (2015) showed that the relationship between idiosyncratic volatility and return is negative only among overpriced stocks; among underpriced stocks, this relationship is positive. To distinguish between overpriced and underpriced stocks, Stambaugh et al. (2015) used an aggregated mispricing score based on 11 well-known equity anomalies. The authors explained the observed variability in risk-return relationships with the arbitrage asymmetry observed in the stock markets—it is easier to buy an undervalued stock than to short an overvalued one.

This study aims to provide an alternative explanation for the phenomenon discovered by Stambaugh et al. (2015). We cast doubt on the arbitrage-based explanation of the idiosyncratic volatility puzzle, indicating that it could be a mere statistical artifact. Stambaugh et al. (2015) used 11 previously documented anomalies and conducted their tests within the US equity

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market where their returns' predictive abilities had been already proven. In consequence, by dividing companies into the underpriced and overpriced categories, they effectively differentiated stocks by their ex-post alphas, i.e., intercepts from a regression equation representing a given asset-pricing model.[1] Thus, to some extent, the results may be influenced by look-ahead bias—the companies are ranked by their future abnormal returns with respect to the factor models. In this paper, we show that the relationship between idiosyncratic volatility and returns is always positive for positive-alpha stocks and negative for negative-alpha stocks, when the alpha is measured as the ex-post abnormal performance. The phenomenon is not driven by any arbitrage—nor behavioral-driven bias, but results purely from the mathematical properties of return distributions.

To prove our point, we perform two exercises. We start with a mathematical proof showing that the patterns observed by Stambaugh et al. (2015) are also present in random samples. Subsequently, we show how this phenomenon translates into portfolio returns. We conduct a battery of Monte Carlo simulations reflecting real-life experience and simulate random returns on the artificial assets for multiple time windows. Using extensive robustness tests, we document that, even in a completely random environment, when the returns are free of any equity anomalies, the relationship between idiosyncratic risk and performance measured by returns will be positive among positive-alpha stocks and negative among negative-alpha stocks.

The rest of the study is organized as follows. Section 2 sets the theoretical basis for the simulation, providing the mathematical proof of the existence of risk-return relationships that are negative among overvalued stocks and positive among undervalued ones. Section 3 outlines the design of our simulation and Section 4 discusses the results. Finally, Section 5 concludes the paper.

2. Theoretical argumentation

Let us consider the seemingly unrelated regression equations (SUR):

$$\boldsymbol{R_a} = \boldsymbol{\alpha} + \boldsymbol{F\beta} + \boldsymbol{E}, \tag{1}$$

where $\boldsymbol{R_a} = [\boldsymbol{R}_1', .., \boldsymbol{R}_n']'$ and $\boldsymbol{R_i} = [\boldsymbol{R}_{i,1}, .., \boldsymbol{R_{i,T}}]'$ is a vector of excess returns on risky assets; $\boldsymbol{\alpha} = [\boldsymbol{\alpha_1'}, ..., \boldsymbol{\alpha_n'}]'$ and $\boldsymbol{\alpha_i} = \boldsymbol{\alpha_i} 1_{\mathbf{T}}$ is a vector of alphas, or mispricing coefficients; $\boldsymbol{F} = \boldsymbol{I_n} \otimes \boldsymbol{R_b}$ where $\boldsymbol{R_b} = [\boldsymbol{f}_1, ..., \boldsymbol{f_K}]$ is the $\boldsymbol{T} \times \boldsymbol{K}$ matrix and $\boldsymbol{f_i} = [\boldsymbol{f}_{\boldsymbol{i},1}, .., \boldsymbol{f_{i,T}}]'$ is the vector of excess returns of factor *i;* $\boldsymbol{\beta} = [\boldsymbol{\beta_1'}, ..., \boldsymbol{\beta_n'}]'$ is the $\boldsymbol{kn}$—vector of factor exposures (loadings), where $\boldsymbol{\beta_i} = [\boldsymbol{\beta}_{1,\boldsymbol{i}}, .., \boldsymbol{\beta_{K,i}}]'$; $\boldsymbol{E} = [\boldsymbol{\varepsilon_1'}, ..., \boldsymbol{\varepsilon_n'}]'$ is a $\boldsymbol{Tn}$—vector of error terms, where $\boldsymbol{\varepsilon_i} = [\boldsymbol{\varepsilon}_{\boldsymbol{i},1}, .., \boldsymbol{\varepsilon_{i,T}}]'$ and it is assumed that: $\boldsymbol{E} \sim \boldsymbol{N}(0, \ \boldsymbol{\Omega})$ and $\boldsymbol{\Omega} = \boldsymbol{\Sigma_\varepsilon} \otimes \boldsymbol{I_T}$.[2]

In market equilibrium, abnormal returns (alphas) from an asset-pricing model are equal to zero. In other words, if the true values of factor exposures capture all differences in expected returns, the alphas $\boldsymbol{\alpha}$ in (1) should be indistinguishable from zero (e.g., Bodie et al., 2013; Fama and French, 2012, 2017). Hence, we assume that:

$$\boldsymbol{\alpha} = 0, \ \text{ so } \ \boldsymbol{E}(\boldsymbol{R_a}) = \boldsymbol{E}(\boldsymbol{F})\boldsymbol{\beta}. \tag{2}$$

Each *i* in the equation in (1) is a multiple regression model. Importantly, Greene (2011) indicates that in the SUR model, when all equations have the same regressors, the efficient estimator is single-equation ordinary least squares (OLS).

Let us now consider the following two regression equations:

$$R_{i,t} = \alpha_i + \boldsymbol{R_{b,t}\beta_i} + \varepsilon_{i,t}, \ \ \varepsilon_{i,t} \sim N\left(0, \ \sigma_i^2\right), \tag{3}$$

$$R_{j,t} = \alpha_j + \boldsymbol{R_{b,t}\beta_j} + \varepsilon_{j,t}, \ \ \varepsilon_{j,t} \sim N\left(0, \ \sigma_j^2\right), \ \ t = 1 \ldots, T, \tag{4}$$

where $\boldsymbol{R_{b,t}}$ is a *t*-row of matrix $\boldsymbol{R_b}$. Let $\hat{\alpha}_i$ and $\hat{\alpha}_j$ be OLS estimators of α_i and α_j, respectively. Our aim is to prove that if $\sigma_i^2 \ < \ \sigma_j^2$, then:

$$E\left(\hat{\alpha}_i | \hat{\alpha}_i > 0\right) < E\left(\hat{\alpha}_j | \hat{\alpha}_j > 0\right). \tag{5}$$

For this purpose, let us notice that the OLS estimators $[\hat{\alpha}_i, \hat{\boldsymbol{\beta}}_{\boldsymbol{i}}']'$ and $[\hat{\alpha}_j, \hat{\boldsymbol{\beta}}_{\boldsymbol{j}}']'$ are normally distributed, i.e.:

$$N\left([\alpha_i, \ \boldsymbol{\beta}_i']', \ \sigma_i^2(\boldsymbol{R'R})^{-1}\right) \ \text{and} \ N\left([\alpha_j, \ \boldsymbol{\beta}_j']', \ \sigma_j^2(\boldsymbol{R'R})^{-1}\right), \tag{6}$$

where $\boldsymbol{R} = [1_{\mathrm{T}}, \boldsymbol{R_b}]$. However, Eq. (2) implies that $\alpha_i = \alpha_j = 0$, so to prove inequality (5), it is sufficient to show that for some OLS estimator of intercept α, if $\hat{\alpha} \sim N(0, \ \sigma^2)$, where σ^2 is a proper element of diagonal matrix $\sigma_i^2(\boldsymbol{R'R})^{-1}$, then $E(\hat{\alpha}|\hat{\alpha} > 0)$ is a linear function of σ. By performing simple derivations, we obtain:

$$E\left(\hat{\alpha}|\hat{\alpha} > 0\right) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{x}{\sigma} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} dx = \frac{\sigma}{\sqrt{2\pi}} \tag{7}$$

[1] Generally speaking, so called "alpha" is a widely-used ex-post measure of investment performance (Bodie et al., 2013). Positive alpha suggests that the portfolio displayed attractive returns in comparison to its underlying systematic risk factors, whereas negative alpha indicate that the portfolio produced poor returns in comparison to its underlying systematic risk factors.

[2] We use bold symbols to represent vectors and matrices.

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