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# One-fund separation in incomplete markets with two assets

Dong Chul Won

School of Business, Ajou University, 206 World cup-ro, Yeongtong-gu, Suwon 16499, South Korea

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## ABSTRACT

This paper provides a necessary and sufficient condition for one-fund separation to occur in incomplete-market economies where finitely many agents with distinct risk aversion and heterogeneous beliefs are allowed to trade two assets. The condition involves joint restrictions on risk aversion, beliefs and asset payoffs. Thus, such joint restrictions may be indispensable for fund separation in incomplete markets, which is in contrast with the preference-based and return-distribution-based approaches. When the condition for one-fund separation holds, agents could behave in equilibrium as if there were a single fund which delivers the aggregate asset payoffs in the economy. Otherwise, agents choose optimal shares in distinct proportions.

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## 1. Introduction

The finance literature has been interested in finding conditions for mutual fund separation. Agents are supposed to hold a portfolio of mutual funds instead of individual assets in economies where mutual fund separation holds in equilibrium. A classical example is the two fund separation of the capital asset pricing model in which agents allocate their wealth between a riskless asset and the market portfolio of risky assets. There are two established approaches to fund separation in the literature: [Cass and Stiglitz \(1970\)](#) and [Ross \(1978\)](#) which characterize conditions for fund separation in terms of utility functions and return distributions, respectively.

This paper provides a distinct perspective into fund separation by introducing a new condition which imposes joint restrictions on preferences, beliefs and asset payoffs. The new condition is necessary and sufficient for one-fund separation in incomplete-market economies where finitely many agents with distinct CRRA (constant relative risk aversion) and heterogeneous beliefs are allowed to trade two assets. The new condition involves joint restrictions on distinct relative risk aversion, heterogeneous beliefs, and asset payoffs. Thus, such joint restrictions may be indispensable for fund separation in incomplete markets, which is in contrast with the preference-based and return-distribution-based approaches. Agents are initially endowed with shares of the two assets, implying the initial endowments are naturally spanned by the asset payoffs.

Fund separation is a time-honored issue in the finance literature. [Cass and Stiglitz \(1970\)](#) characterize the class of utility functions which possess the separation property that optimal portfolios consist of a limited number of mutual funds. In contrast, [Ross \(1978\)](#) provides necessary and sufficient conditions on return distributions for fund separation to hold independently of preferences. [Cass and Stiglitz \(1970\)](#) and [Ross \(1978\)](#) discuss fund separation in the case where agents

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*E-mail address:* [dcwon@ajou.ac.kr](mailto:dcwon@ajou.ac.kr)<http://dx.doi.org/10.1016/j.frl.2017.09.003>

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have homogeneous beliefs. In particular, [Ross \(1978\)](#) examines one-fund separation from the distributional viewpoint. The joint condition of the current paper for one-fund separation is specialized in the case with two assets where agents have distinct homothetic preferences and heterogeneous beliefs. [Detemple and Gottardi \(1998\)](#) study fund separation and aggregation issues in incomplete markets where agents have identically locally homothetic preferences and spanned endowments. However, they do not cover the case with heterogeneous beliefs. [Schmedders \(2007\)](#) investigates the two-fund separation property in an infinite horizon economy with dynamically complete markets. [Schmedders \(2007\)](#) shows that two-fund separation holds if a consol is available but may fail if the console is replaced by a series of one-period bonds.

## 2. Model

We consider a simple two-period finance economy with finitely many agents indexed by  $i \in \mathcal{I} = \{1, \dots, I\}$  who consume a single good both in the first period and the second period. The first period is denoted by state 0 and uncertainty of the second period consists of finitely many states indexed by  $s = 1, \dots, S$  with  $S \geq 3$ . Let  $\mathcal{S}$  denote the set  $\{1, \dots, S\}$  of the second-period states and  $\mathcal{S}^a$  the augmented set  $\{0\} \cup \mathcal{S}$  of the  $S+1$  states. Since consumption arises in each  $s \in \mathcal{S}^a$ , a consumption plan is denoted by a point in  $\mathbb{R}^{\mathcal{S}^a}$ .

There are two securities  $j = 1, 2$  which are traded in each period. Since  $S \geq 3$ , asset markets are incomplete. Asset  $j$  pays  $r_s^j$  units of the single good in each state  $s \in \mathcal{S}^a$ . Let  $r^j$  denote the payoff vector  $(r_0^j, \dots, r_S^j) \in \mathbb{R}^{\mathcal{S}^a}$  of asset  $j$  and  $r_s = (r_s^1, r_s^2)$  the payoff vectors of the two assets in state  $s \in \mathcal{S}^a$ . We introduce payoff matrices  $R^a$  and  $R$  where  $R^a$  is a  $(S+1) \times 2$  matrix with  $r^j$  as its  $j$ th column and  $R$  is a  $S \times 2$  matrix defined by

$$R = \begin{bmatrix} r_1^1 & r_1^2 \\ \vdots & \vdots \\ r_S^1 & r_S^2 \end{bmatrix}.$$

Agent  $i$  is initially endowed with shares  $\bar{\theta}^i = (\bar{\theta}_1^i, \bar{\theta}_2^i)$  of the assets. The outstanding shares of each asset is normalized to 1. Thus, it holds that  $\sum_{i \in \mathcal{I}} \bar{\theta}_j^i = 1$  for each  $j = 1, 2$ .

Preferences of agent  $i$  over the consumption set  $X \equiv \mathbb{R}_+^{\mathcal{S}^a}$  are represented by a utility function

$$u_i(x^i) = v_i(x_0^i) + \rho \sum_{s \in \mathcal{S}} \pi_s^i v_i(x_s^i), \quad x^i \in X.$$

Here  $\rho$  indicates a time discount or a weight between the current utility and the expected future utility while  $\pi_s^i$  a subjective probability of agent  $i$  that state  $s$  occurs in the second period. For an asset price  $q = (q_1, q_2)$ , agent  $i$  faces the following choice problem

$$\begin{aligned} \max_{(x, \theta) \in X \times \mathbb{R}^2} & u_i(x) \\ \text{s.t.} & x_0 \leq -q \cdot (\theta - \bar{\theta}^i) + r_0 \cdot \bar{\theta}^i, \\ & x_s \leq r_s \cdot \theta \quad \text{for all } s \in \mathcal{S}. \end{aligned} \quad (1)$$

We make the following assumptions.

**Assumption 1.** Each function  $v_i$  is concave and homogenous of degree  $k_i < 1$ , i.e., for each  $\lambda > 0$  and  $y > 0$ ,  $v_i(\lambda y) = \lambda^{k_i} v_i(y)$ .

**Assumption 2.** The payoff matrix  $R$  has full rank and  $R^a$  satisfies  $R^a \cdot \bar{\theta}^i \gg 0$ .<sup>1</sup>

The homogeneity of [Assumption 1](#) implies that  $v_i(y) = y^{k_i} v_i(1)$  for all  $y > 0$ . By the concavity of  $v_i$ , it holds that if  $v_i(1) > 0$ ,  $0 \leq k_i < 1$ , and if  $v_i(1) < 0$ ,  $k_i \leq 0$ . When we can put  $k_i = 1 - \gamma_i$ ,  $\gamma_i > 0$  indicates a constant relative risk aversion. When  $k_i < 1$ ,  $v_i$  has infinite marginal utility at 0 which makes agents choose positive consumption in each state. The second assumption ensures that each agent can make positive consumptions in each state under autarky.

**Remark 1.** The two-period finance economy where agents are initially endowed with securities can be transformed into a two-period economy of the general equilibrium literature where agents are initially endowed with consumption goods by rewriting the second-period budget constraints as

$$x_s - r_s \cdot \bar{\theta}^i \leq r_s \cdot (\theta - \bar{\theta}^i), \quad s \in \mathcal{S}.$$

The second part of [Assumption 2](#) ensures that agents have the positive initial endowments  $\{r_s \cdot \bar{\theta}^i, s \in \mathcal{S}^a\}$  which are spanned by the asset payoffs in the latter economy. Thus, the second part of [Assumption 2](#) implies a strong survival condition of the general equilibrium literature which guarantees the existence of equilibrium under standard conditions such as continuity and convexity of preferences. It also yields

$$R^a \cdot \left( \sum_{i \in \mathcal{I}} \bar{\theta}^i \right) = R^a \cdot \mathbf{1}_2 \gg 0, \quad (2)$$

<sup>1</sup> For two points  $x, y$  in  $\mathbb{R}_+^{\mathcal{S}^a}$ ,  $x \geq y$  if  $x - y \in \mathbb{R}_+^{\mathcal{S}^a}$ ,  $x > y$  if  $x \geq y$  and  $x \neq y$ , and  $x \gg y$  if  $x - y \in \mathbb{R}_+^{\mathcal{S}^a}$ .

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