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Note

A simple characterization of responsive choice ☆

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ABSTRACT

We provide several characterizations of q-responsive choice functions, based on classical axioms of matching theory and revealed preference theory.

1. Introduction

We characterize choice functions which arise in matching theory. Our particular interest is in choice functions for which there is a rational (strict) preference relation that always selects the q best elements whenever available. Such choice functions are termed q-responsive. In the context of schools with preferences over students, for example, such a choice function would be one for which the school admits the q highest-ranked students.

We seek to understand the behavioral underpinnings of this choice theoretic model when the preference itself is not directly observable. To this end, we provide several such characterizations. Notably, we provide characterizations for choice functions which operate on the power set of a set of alternatives, but also on smaller sets of budgets.

On the power set, we can imagine a choice function that always selects q alternatives whenever there are q available, and, otherwise, selects all of the alternatives. This concept is termed q-acceptance. It turns out that, together with q-acceptance, it is enough to impose an axiom of Jamison and Lau (1973), the weaker axiom of revealed preference (WrARP).[2] This axiom states that for any pair of partners, x, y, if x is chosen when y is available and y is not chosen, then it can never be that y is chosen when x is available and x is not chosen. We show that a choice function is q-responsive if, and only if, it is q-acceptant and satisfies WrARP (Theorem 1). This result is a direct generalization of the classical result that a single-valued choice function satisfying the weak axiom of revealed preference is classically rationalizable.

We also show how to utilize a weakened version of WrARP, applying only to sets of cardinality $q+1$, together with a familiar axiom from matching theory (substitutability) to characterize the same family (Theorem 2).

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1 Some of the results in this paper initially appeared in the working paper version of authors' paper titled "Choice and Matching."

2 A referee pointed out to us that this axiom was first formally introduced in Jamison and Lau (1973). It is also used as Axiom 3 in Fishburn (1975) and as the weakened weak axiom of revealed preference in Ehlers and Sprumont (2008).

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A different variation of WrARP was discussed in Aleskerov et al. (2007) or Tyson (2008), and used to axiomatize a form of satisficing behavior.[3] For lack of better terminology, we call it the weakened strong axiom of revealed preference. The axiom roughly rules out strict revealed preference cycles of any length. Aleskerov et al. (2007) and Tyson (2008) use the axiom to characterize choice functions for which there is a rational preference, and for any set of alternatives, the choice function always selects some "at least as good as set" from the set. Hence, if it selects an alternative, it must select any present alternative which is at least as good as the one selected. It is a short step to add q-acceptance to fully characterize q-responsive choice functions here.

In fact, as is made clear in Ehlers and Sprumont (2008) (see also Chambers and Echenique, 2016, Theorem 2.17), the class of choice functions satisfying WrARP can be exhaustively characterized as those for which there is a (potentially intransitive) preference, and for any set, the choice function selects some "at least as good as set."

A precedent to this note is the work of Eliaz et al. (2011), who characterize the same rules by means of different axioms.[4] Both characterizations use q-acceptance. In our characterization, we also use WrARP. In their characterization, they use two other axioms. We show that WrARP is not logically related to their two axioms with examples. But obviously under q-acceptance WrARP is equivalent to the two axioms that they use. Furthermore, their work provides a characterization only for choice functions defined on the power set as in our Theorem 1. Our axiomatization, based on axioms which are standard in the literature, allows us to establish a characterization on choice functions which are not defined on the power set, in an exercise analogous to Richter (1966). See also DeClippel and Rozen (2014) for a recent investigation of this idea in modern behavioral models, as well as Chambers et al. (2014).

There are some other closely related works. In our earlier work, we study a more general class of choice functions in the context of matching (Chambers and Yenmez, 2017). The recent paper of Doğan et al. (2017), which follows up on our results, provides a parametric characterization of q-responsive choice in a setting where choice functions take q as a parameter. They provide a characterization demonstrating how to ensure that the preference relation used for each q is the same. Their characterization builds on our result by introducing axioms referencing the variation in q. In another interesting contribution, Barberà and Neme (2015) characterizes those single-valued choice functions which are *selectors* of some q-responsive choice function.

Finally, we wish to emphasize that though our results are framed in terms of matching theory, they can be applied more broadly. Conceptually, the WrARP condition here might be viewed as ruling out certain types of "complementarities" across the chosen elements. For example, suppose that the choice function chooses teams of baseball players from amongst a set of potential players. Then we would ask that the function be 9-acceptant. However, the players obviously could not be directly ranked by a single preference relation. A good pitcher would be a poor substitute for a strong hitter, though both are desirable. In such a situation, our results would not apply. See Barberà et al. (2004) for a discussion of ranking sets.

2. The model

Suppose $\mathcal{X}$ is a set of alternatives and $\mathcal{P}(\mathcal{X}) = 2^{\mathcal{X}}$ is the powerset of $\mathcal{X}$. Let $\Sigma \subseteq \mathcal{P}(\mathcal{X})$ be a set of *budgets*. A *choice function* is a mapping $C : \Sigma \to \mathcal{P}(\mathcal{X})$ such that

- for every $X \in \Sigma$, $C(X) \subseteq X$ and
- for every $\emptyset \neq X \subseteq \mathcal{X}$, $C(X) \neq \emptyset$.

We require the chosen set to be non-empty if there is at least one alternative available.

A *preference relation* $\succeq$ on $\mathcal{X}$ is a binary relation on $\mathcal{X}$ that is complete, transitive, and antisymmetric.[5] Let $q \in \{1, 2, \ldots\}$ be an integer.

We first introduce the following class of choice functions.

Definition 1. Choice function C is **q**-*responsive* if there is a preference relation $\succeq$ such that for every $X \in \Sigma$ such that $|X| \leq q$, $C(X) = X$, and for every $X \in \Sigma$ such that $|X| > q$, then $C(X)$ is defined as $C(X) = \{x_1^*, \ldots, x_q^*\}$, where $x_1^* = \arg\max_{\succeq} X$, and for all $i = 2, \ldots, q$, $x_i^* = \arg\max_{\succeq} X \setminus \{x_1^*, \ldots, x_{i-1}^*\}$. Choice function C is *responsive* if it is q-responsive for some q.

In other words, C is q-responsive if there is a preference relation for which C selects the highest q alternatives, whenever available. Such choice rules are used in matching literature including the seminal work of Gale and Shapley (1962).

Definition 2. Choice function C is *substitutable* if for every $X, Y \in \Sigma$ and $x \in X \subseteq Y$, $x \in C(Y)$ implies $x \in C(X)$.

[3] In our context, q-responsive rules are a form of satisficing behavior. See Chambers and Echenique (2016) for details.

[4] To be more precise, they provide a characterization of q-responsive choice rules for $q = 2$. However, in a later comment, they discuss how the proof can be generalized for any q. See our discussion below.

[5] Complete: For all $x, y \in \mathcal{X}$, $x \succeq y$ or $y \succeq x$. Antisymmetric: For all $x, y \in \mathcal{X}$, $x \succeq y$ and $y \succeq x$ implies $x = y$. Transitive: For all $x, y, z \in \mathcal{X}$, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$.