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# Keep on fighting: The dynamics of head starts in all-pay auctions ☆

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ABSTRACT

We investigate a model of a series of contests in which a contestant's past and present success gives a head start over a rival in the future. How this advantage from winning affects contestants' efforts, whether the laggard gives up or keeps on fighting, and how the head start develops over time, are key issues. We find that the expected effort of the laggard will always be higher than the rival at some stage in the series of contests, and this is most likely to happen when at a large disadvantage or at a late stage in the series.



## 1. Introduction

Winning a competition may result not only in a prize, but also in an advantage in subsequent competitions. Consider, for example, competitions for research grants. While the successful applicant for a grant may harvest all the direct benefits that the research money awarded provides, there may also be an extra benefit from winning: carrying out the research that the original grant facilitates makes for increased chances to win in future grant competitions. In this way, an early competition for a prize implies that there will be advantaged and disadvantaged participants in subsequent competitions: winning an early contest gives you a head start in later contests. Questions are then how contestants' incentives to put in effort in such sequential competitions vary over time as successes and failures are recorded, and how these incentives interact in the shaping and development of head starts.

In order to understand the dynamics of this kind of competition, we develop in this paper a two-player model of a series of contests where, in each stage contest, there is a prize to win of common value. A player wins a stage contest by having a larger score than the opponent, where a player's score consists of effort in the current contest and the number of contests that the player has won so far; we modify the number of wins by a parameter that captures the importance of winning,

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and this is assumed identical for the players.[1] Hence, the player with the larger number of wins has a head start in any stage contest since he can win now even when investing less effort than the rival. We point out two forces that interact in explaining contestants' incentives across time. On one hand, there is a large incentive to be the player with the head start since this can save effort costs in the future without necessarily reducing the probability of success. However, the head start creates an asymmetry which lowers both players' incentives to put in effort, but more so for the disadvantaged player – the laggard. This is because the head start enables the advantaged player – the leader – to lay back a bit and still stand a good chance to win again, so that also the laggard pulls back somewhat.

On the other hand, there is an additional value of being the leader. A new win means he will also be a leader in the future, while a win for the laggard will at best even the score. The difference in value between winning and losing at each stage is therefore larger for the leader than for the laggard. A win for the laggard will at best even the score (with an expected payoff of zero), and a loss means that the laggard is still at a disadvantage (with the same expected payoff of zero). This dampens the laggard's incentives to put in effort. However, the value of winning and continuing as leader falls over time in a game of finite length, simply because there are fewer future contests left; hence, even the leader will put in less effort at some stage in the contest sequence. This encourages the laggard who, towards the end of the sequence of contests, will be the high performer. The balance of these effects – and the interplay between them – cannot be captured in a one-shot game with a head start.

Above, we mentioned the advantage that winning a current research grant can convey on the winner in future competitions. It is the actual fact of winning that creates the advantage here; whether an application is a little better or much better than that of the opponent is not relevant. Winning an early grant, and being able to carry out a research program, enhances the chance to win again in the competition for later grants. Winning gives a player a head start in the next contest. This can also be expected to occur in a number of other contexts. In sales-force management, it is customary to give awards to the Seller of the Month and the like. And in such sales forces, it is not uncommon for the more successful agents to be given less administrative duties, better access to back-office resources, more training than the less successful, and better territories; see, e.g., Skiera and Albers (1998), Farrell and Hakstian (2001), and Krishnamoorthy et al. (2005). Being a winning salesperson gives a head start in later contests.

In our model, winning creates a head start advantage. Another strand of the literature considers that winning agents have access to different prizes than less successful ones (Megidish and Sela, 2014). Also a psychological momentum may accrue to winners (Krumer, 2013). Experiments carried out by Eriksson et al. (2009) indicate that laggards keep on fighting. In particular, the authors compare laggards who get feedback, i.e., are informed that they are lagging, with ones who do not get such feedback and find that informed laggards are more prone to put in effort. This finding fits well with our theoretical predictions if we view winning a competition in these experiments as getting a momentum before later competitions.

The sequence of contests that we model in this paper gives, as noted, rise to the creation of a leader and a laggard by which the winner of an early contest gets an advantage in the next one. Such dynamic win effects, in various forms, are also discussed by Krumer (2013), Megidish and Sela (2014), and Clark et al. (2018). Krumer's (2013) discussion is in the context of a race, whereas the other two papers are on sequences of Tullock contests. These contributions are confined to analyzing sequences of two stage competitions. In the present study, on the other hand, we allow for longer sequences, and the stage contest is an all-pay auction. With long sequences of contests, we are able to discuss how the interaction of the leader and the laggard's incentives develop over time.

Leaders and laggards also feature in races, i.e., best-of-$t$ contests, where the overall winner is the first to win $t$ stage contests; see Harris and Vickers (1987) for an early analysis and Konrad (2009) for an overview.[2] The winner of the first stage of a race becomes the leader in the second, in the sense of having fewer stages left to complete the game. This leader has a much firmer grip on the rest of the game than the leader has in our context. Results differ in the two set-ups, not surprisingly. While the laggard is strongly discouraged in a race, he is much more interested in staying and keep on fighting in our setting.

Strumpf (2002), Konrad and Kovenock (2009, 2010), Fu et al. (2015), and Konrad (2018) show various ways in which the discouragement of the laggard can be mitigated in a race. In Strumpf (2002), this happens when the contests most valuable to the laggard are late in the race; in Konrad and Kovenock (2009), it happens because of the introduction of stage prizes; in Konrad and Kovenock (2010) because of the introduction of uncertainty; in Fu et al. (2015) because of the introduction of team competition; and in Konrad (2018) because of players having fixed budgets over the sequence of contests. In these papers, there is no dynamic win effect. While the discouragement of the laggard is mitigated, he never exerts the higher expected effort, as he eventually does in our analysis. Of these papers, Konrad and Kovenock (2009) is particularly closely related to the present work, and we therefore include a more detailed discussion of this relationship in Section 6 below.

Gelder (2014), on the other hand, shows, in his analysis of a race, that a combination of punishment from loss and discounting creates a scope for what he calls "last stand" behavior. This resembles our result that the laggard exerts the higher expected effort in the final period, and often also earlier. Gelder thus establishes an equilibrium outcome similar to

[1] Clark and Nilssen (2018) consider a series of two all-pay auctions in which winning the first contest can give the players different head starts in the second.

[2] Another interesting multi-period situation creating a leader and a laggard is the incumbency competition, where the winner of contest $t$ becomes the leader at contest $t+1$; see, e.g., Ofek and Sarvary (2003) and Mehlum and Moene (2006, 2016).