Games and Economic Behavior 109 (2018) 99–103

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Note

A new look at the classical Bertrand duopoly

Rabah Amir [a,*], Igor V. Evstigneev [b]

[a] Department of Economics, University of Iowa, Iowa City, IA 52242-1994, USA
[b] Department of Economics, University of Manchester, Oxford Road, M13 9PL, UK

ARTICLE INFO

Article history:
Received 20 February 2017
Available online 15 December 2017

JEL classification:
C72
L10

Keywords:
Price competition
Uniqueness of Nash equilibrium
Weakly dominated strategy
Non-monotonic demand

ABSTRACT

The paper provides necessary and sufficient conditions for the uniqueness of pure-strategy Nash equilibrium in the standard Bertrand duopoly with a homogeneous product. The main condition is elementary, easy to interpret, and nests all known sufficient conditions in the literature.

1. Introduction

Although written in the form of a book review, Bertrand's (1883) critique of Cournot's (1838) oligopoly turned out to form the most widely used model of price competition. Indeed, nowadays, the Bertrand duopoly model is one of the cornerstones of introductory microeconomics and game theory. The "Bertrand paradox" usually refers to the unusual equilibrium outcome of perfect competition in a market with just two firms.[1] The strategies whose implementation leads to this outcome prescribe to set the minimal prices (equal to the marginal costs), resulting in zero profits. The fact that such strategies form a Nash equilibrium is a simple observation, and actually does not require any assumptions on the model. However, uniqueness holds only under additional assumptions, and its proof requires some quite elementary but novel arguments.

To the best of our knowledge, the literature provides no necessary and sufficient conditions for uniqueness of pure-strategy Bertrand equilibrium. In this note, we formulate such a condition in terms of static collusion being unsustainable, an intuitive assumption. In addition, we also provide easily verified sufficient conditions for this key assumption in terms of standard properties of demand. This finding seems to be the first result of this kind after more than a century of studies on the Bertrand duopoly model and its variants.

Owing to a number of special game-theoretic features, the classical Bertrand duopoly has generated broad interest extending far beyond oligopoly theory and modern industrial organization. The first feature of interest is that the Bertrand

* Corresponding author.
E-mail addresses: rabah-amir@uiowa.edu (R. Amir), igor.evstigneev@manchester.ac.uk (I.V. Evstigneev).
URLs: https://tippie.uiowa.edu/people/rabah-amir (R. Amir), http://www.evstigneev.net (I.V. Evstigneev).

[1] It is well known that the Bertrand paradox is in many ways not very robust. For the case of convex costs, see Dastidar (1995). For the Bertrand–Edgeworth version with unequal unit costs, see Deneckere and Kovenock (1996). For product differentiation and endogenous timing, see Amir and Stepanova (2006).

https://doi.org/10.1016/j.geb.2017.12.010
0899-8256/

game is the oldest representative of the class of classical games with naturally discontinuous payoff functions.[2] In addition, the payoff discontinuity along the price diagonal is extreme: the low-priced firm grabs the entire market and the rival gets nothing.[3] Another key feature of the classical Bertrand game is that the unique Nash equilibrium is in weakly dominated strategies. Indeed, pricing at marginal cost yields zero profit to the two firms. Finally, the Nash equilibrium payoffs correspond to the individually rational payoffs of the firms, or their maxmin payoffs, which makes the infinitely repeated version of the game a convenient and robust model for studying tacit collusion via the Folk Theorem.

A key reason the Bertrand model has generated broad interest beyond oligopoly theory is the fact that other, a priori unrelated, games in experimental economics and game theory turned out to share these properties in one form or another. These games include the Guess-The-Average game (Moulin, 1986), which, along with various extensions, has spurned an extensive literature in experimental economics, see e.g. Nagel et al. (2017). This is a constant-sum game wherein players pick numbers in $[0, 100]$ and the one closest to $2/3$ of the average wins a fixed prize (with ties sharing the prize equally), and the others get 0. In the two-player version, 0 is also a weakly dominant strategy (Grosskopf and Nagel, 2006). The unique Nash equilibrium, which calls for every player to pick 0, garnered little support in laboratory experiments, although play unraveled towards it. Yet, a discretized and stylized version of the Bertrand game confirmed the usual equilibrium, more so with more than two players (Dufwenberg and Gneezy, 2000).

Although pertaining to a different social situation, the Traveler's Dilemma (Basu, 1994) has a similar structure, with the key modification that the value of "undercutting the rival" is smoothed out to a small reward, in contrast to the extreme gap characterizing the other two games. Although the Nash equilibrium calls for the players to pick the lowest number and share the prize, laboratory behavior displayed significant departures from this robust theoretical prediction (Capra et al., 1999).

One possible implication of the present study of Bertrand duopoly is that it might shed some light towards a systematic study of the aforementioned general class of games and their variants. Another interesting implication is that the downwards monotonicity of the demand function is not relevant to the existence or uniqueness of the Bertrand–Nash equilibrium (we shall return to this point below). As a final remark, it is worth noting that, by offering a complete characterization of uniqueness of Nash equilibrium, our elementary result has no antecedents in game theory, in that uniqueness is typically guaranteed with sufficient conditions that are often quite far from necessary. Indeed, uniqueness almost invariably follows from a global contraction-type argument.

2. The main result

We consider a version of the Bertrand duopoly model with a homogeneous product. There are two profit-maximizing firms 1 and 2 producing a homogeneous good in a market whose demand function is given by $D(p) \geq 0$ $(p \geq 0)$. The cost, $c \geq 0$, per unit produced is the same for both firms. The firms simultaneously set their prices p_1 and p_2. Sales for firm i are then given by

$$D_i(p_1, p_2) = \begin{cases} D(p_i), & \text{if } p_i < p_j; \\ \frac{1}{2} D(p_i), & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

The firms' profit functions are

$$\pi_i(p_1, p_2) = (p_i - c) D_i(p_1, p_2), \ i = 1, 2.$$

We assume that the firms never set prices that are less than c: if $p_i < c$, then firm i cannot have a strictly positive profit $\pi_i > 0$. Thus we have a game with payoffs $\pi_1(p_1, p_2)$, $\pi_2(p_1, p_2)$ and the strategy set $P_c = [c, \infty)$ for both players. The game is symmetric: $\pi_2(p_1, p_2) = \pi_1(p_2, p_1)$.

Since we consider only those prices p_i that satisfy $p_i \geq c$, it is sufficient to assume that $D(p)$ is defined only for $p \geq c$.

We are interested in (pure-strategy) Nash equilibria of the above game (Bertrand–Nash equilibria), i.e., pairs of prices (p_1^*, p_2^*), $p_i^* \geq c$, such that

$$\pi_1(p_1^*, p_2^*) \geq \pi_1(p_1, p_2^*) \text{ and } \pi_2(p_1^*, p_2^*) \geq \pi_2(p_1^*, p_2)$$

for all $p_1, p_2 \geq c$.

The next result is well-known (though note the absence of the usual assumptions).

[2] The payoff functions are not even upper semi-continuous in the prices, so the firms' reaction curves are not well defined. Thus the Bertrand duopoly does not fit the usual classes of games with discontinuous payoffs (Reny, 2016). In addition, the payoffs are not quasi-concave in own action, so the results from that literature do not apply. Nonetheless, an exception is Prokopovych and Yannelis (2017), who derive a general result that includes existence in the Bertrand model as a special case.

[3] As a consequence, the Bertrand game belongs to a family of games for which a tie breaking rule is a necessary part of the definition of the game, and its specification is often a critical part of its solution.