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How to split the pie: Optimal rewards in dynamic multi-battle competitions[☆]

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ABSTRACT

Multi-battle competitions are ubiquitous in real life. In this paper, we examine the effort-maximizing reward design in sequentially played multi-battle competitions between two players. The organizer has a fixed prize budget, and rewards players contingent on the number of battles they win in a three-battle contest. A full spectrum of contest technologies in the Tullock family is accommodated. We find that the optimal design varies with the discriminatory power of the contest technology. In particular, when it is in the low range, winner-take-all is optimal. For the intermediate range, as discriminatory power increases, the optimal prize structure evolves continuously from winner-take-all to the proportional-division rule due to the need to mitigate the growing momentum/discouragement effect. For the high range, a wide span of prize structures extracts full surplus and is thus optimal. Several robustness checks confirm that mitigating the momentum/discouragement effect is essential for effort-maximizing prize design in dynamic multi-battle contests.

1. Introduction

Dynamic multi-battle contests are ubiquitous. Many economic and social competitions, including research and development races, lawsuits/litigation, bidding for procurement contracts, policy debates (e.g., the U.S. presidential debates), legislative, lobbying, electoral campaigns, and sports can be viewed as contests in which opposing parties expend nonrefundable, costly effort to compete in multiple battles.[1] In such competitions, the final winner of the overall contest, as well as contestants' rewards, is typically determined by the outcomes of *all* battles instead of a *single* battle. For example, in a widely adopted winner-take-all best-of-$(2n + 1)$ contest, a party wins the contest and takes the entire prize if and only if it wins the majority of the battles.

There exists a wide range of diversity in the prize structures of multi-battle contests. Both grand and intermediate prizes are commonly adopted. Often, the final winner is determined according to the above-mentioned majority rule and receives the whole prize. Two-party political campaign competitions (e.g., to gain control of a legislature) and Democratic and Republican primaries to nominate candidates for the U.S. presidential election have long been viewed as winner-take-all multi-battle contests (e.g., Snyder, 1989; Klumpp and Polborn, 2006; Fu et al., 2015; Boyer et al., 2017). On the other hand, battle winners are often awarded intermediate prizes, and such prize allocation is usually contingent on the number of component battles each player wins. This type of prize structure is prevalent in labor tournaments, including sports. The Fédération Internationale de Volleyball (FIVB) World League (for men) and World Grand Prix (for women) are examples of intermediate prizes for each single match. In the group stage of the tournament, each match is a best-of-five game; since 2010, the winning team earns 3 match points and the losing team receives 0 if the final set score is either 3-0 or 3-1. The winning team earns 2 match points and the losing team wins 1 if the final set score is 3-2 (c.f. Jiang, 2014).

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[1] Please refer to Konrad and Kovenock (2009), Konrad (2009), and Kovenock and Roberson (2012), among many others, for examples of multi-battle contests.

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These practices demonstrate how prize allocations can be contingent on the contest outcome, i.e., the number of battles each party wins. Interesting questions thus arise: What drives different choices of prize-allocation rules? In particular, in which situations should the allocation rule solely rely on the performance aggregated over all battles—i.e., the final winning status of the whole contest—and in which situations should the players be awarded separately for each individual battle, but not on their aggregate performance? How do these choices relate to prevailing contest technologies? How does the choice of prize structure evolve as the contest technology becomes less or more discriminatory?

In this paper, we aim to provide a possible answer to these questions from the perspective of effort elicitation by a contest organizer who can flexibly reward contestants based on their numbers of winning battles.[2] For this purpose, we study the optimal contingent prize-allocation rule that elicits the maximum aggregate effort in a sequential-play multi-battle contest between two risk-neutral players with unit marginal effort cost. In every component battle, both players observe the outcomes of previous battles and exert effort simultaneously. We allow a full spectrum of contest technology in the Tullock family to model component battles, which are indexed by the discriminatory power (r) of the corresponding contest success function. The contest organizer has a fixed budget (normalized as 1) to fund nonnegative prizes for competing players. She has the flexibility of fully allocating the budget contingent on battle outcomes, i.e., the wins that each party secures, subject to a monotonicity condition under which the more battles a party wins, the larger their share of the prize.

We fully characterize the optimal contingent prize allocation for every positive discriminatory power $r(>0)$ in a sequentially played three-battle contest.[3] We find that the optimal prize allocation rule crucially depends on the discriminatory power r, which measures the importance of a player's effort in determining his winning probability. A higher discriminatory power r means that the chance of winning is determined more by players' effort than by other random factors (c.f. Fu and Lu, 2012a). We find that when r is low, a winner-take-all best-of-three contest is optimal; when r falls in an intermediate or high range, the optimal design takes the form of a best-of-three contest with both a contest prize to the grand winner and uniform battle prizes to battle winners. In particular, in the intermediate range, the battle prize increases from 0 to 1/3 as r increases, i.e., the optimal prize structure evolves from winner-take-all to the proportional division rule as r increases in this range. In the high range, a whole span of battle prizes, ranging from winner-take-all to the proportional division rule, is equally optimal.

The economics and intuitions behind these characterizations can be illustrated as follows. For convenience, we use $v(n)$ to denote the prize awarded to a player winning $n \in \{0,1,2,3\}$ battles. It is natural that $v(0) = 0$ (and thus $v(3) = 1$) is necessary to elicit maximum effort from players, since rewarding a player without a single win dampens players' incentive. The more interesting and intricate tradeoff lies in the balance between the prizes for a single win and two wins. Any eligible prize profile with $v(0) = 0$ and $v(3) = 1$ is equivalent to the combination of a grand contest prize v_g for the grand winner who wins at least two battles, and a uniform battle prize v_b for the winner of each battle, where $v_b = v(1)$ and $v_g = 1 - 3v(1)$.[4] We can thus focus on the tradeoff between battle prizes and contest prize. An increase in battle prize v_b comes with a three-time drop in contest prize v_g. Because of the fixed budget, we must evaluate which prize contributes more effectively to effort elicitation.

High battle prizes raise players' effective prize spreads in component battles, and therefore increase players' effort supply in each component battle. In addition, high battle prizes can reduce the well-established momentum/discouragement effect in sequentially played multi-battle contests and balance the second-stage contest. The "strategic momentum effect" or "discouragement effect," as first identified by Harris and Vickers (1987), says that one's (perhaps purely accidental) early lead would allow him to attain easy wins in the future, as it forces his lagging opponent to concede prematurely. One extreme example of battle prizes is the proportional division prize allocation rule, in which each battle winner wins one-third of the budget, and thus the momentum/discouragement effect completely disappears.

A high grand prize raises players' effort significantly in the first battle, as well as in the third stage when each player wins one battle. However, it can also accelerate the end of the competition for two reasons. For example, consider a winner-take-all prize structure in which the grand contest prize is set at its maximum. In a situation in which each player has won the first two battles, neither player has incentive to fight in battle 3, since no further prize is provided. Moreover, winner-take-all strengthens the momentum/discouragement effect, which means that the winner of the first battle has higher incentive and higher chance to win battle 2, due to the desirable grand prize, and therefore the contest will more likely end after the first two battles.

The discriminatory power r plays a crucial role in determining the optimal tradeoff between grand prize and battle prizes. In a battle, the higher the discriminatory power r, the more effective the effort will be for determining the winner. When discriminatory power r stays low, the momentum/discouragement effect is weak. Therefore, with high likelihood, the contest will not end soon even under a winner-take-all structure. In this case, the grand prize is more likely to elicit larger effort than battle prizes, which leads to the optimality of a maximum grand prize and zero battle prizes. As the discriminatory power r moves into the intermediate range, the momentum/discouragement effect becomes stronger. To balance the second-stage contest and lengthen the competition, it is optimal to provide a positive battle prize that increases with r. When r moves into a higher range, i.e., $r \geq 2$, players react so sensitively to prizes that their rents are fully dissipated under any eligible prize structure.

Our main insight is that mitigating the momentum/discouragement effect is essential for effort-maximizing prize design in dynamic multi-battle contests. For the purpose of a robustness check, we conduct further analysis by simulations in several extended settings. We expand the set of feasible prize structures by accommodating an unexhausted budget, nonmonotonic prizes, and history-contingent prizes, and we investigate contests with two asymmetric players and contests with 5 battles. The numerical results confirm our insight in the main analysis.

Our paper primarily belongs to the well-established literature on multi-battle contests. Environments in which battles are contested sequentially have been analyzed by Harris and Vickers (1987), Ferrall and Smith (1999), Klumpp and Polborn (2006), Konrad and Kovenock (2009, 2010), McFall et al. (2009), Malueg and Yates (2010), Sela (2011), and Gelder (2014), among others. Harris and Vickers (1987) study a multi-battle patent race. Klumpp and Polborn (2006) model U.S. presidential primaries as a multi-battle dynamic contest between two candidates. Malueg and Yates (2010) study players' strategic effort supply in best-of-three contests and test their theoretical predictions empirically using tennis data. All of these studies identify the strategic momentum/discouragement effect in dynamic multi-battle contests. Konrad and Kovenock (2009) completely characterize the unique subgame perfect equilibrium in multi-battle contests with intermediate prizes, in which component contests are modeled as all-pay auctions. They find that even a large lead by one player may not fully discourage the other when a component battle awards a positive intermediate prize.[5] Konrad and

[2] Gradstein and Konrad (1999) state that "contest structures result from the careful consideration of a variety of objectives, one of which is to maximize the effort of contenders."

[3] In the main analysis, the organizer only has the flexibility to reward players contingent on the number of battle wins. In our robustness checks in Section 4, we will allow an unexhausted budget, nonmonotone prizes, and history-contingent prizes.

[4] This equivalence is established in Section 3.1.2.

[5] Irfanoglu et al. (2011) and Mago and Sheremeta (2012) test these theoretical implications experimentally.