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Modeling and simulations of the amplitude–frequency response of transmission line type resonators filled with lossy dielectric fluids

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ABSTRACT

Stub resonators can be used to assess the dielectric properties of fluids. The resonance frequencies, determined from the amplitude versus frequency (AF) response of such resonators, are mainly determined by the permittivity of the fluid while damping arises from dielectric losses. Even though this methodology has been extensively reported in the literature, without almost any exception these studies refer to (near) ideal behavior regarding for example, geometry and negligibly low conductivity of the fluid studied. Online stub resonator-based sensors (i.e., flow-through) in use for industrial applications, however, quite often suffer from high dielectric losses, non-ideal material choice of the conductors from an electrical point of view and unconventional resonator geometry. Therefore, in order to ensure correct data interpretation, a straightforward model accounting for the effects of dielectric losses, conductor losses (skin effect) and impedance mismatches on the AF response is highly desirable. In addition, such a model can help to optimize future sensor designs. Here, we present a lumped parameter model, essentially based on telegrapher's equations, that accounts for the skin effect, dielectric losses and impedance mismatches between the transmission lines to the resonator and the resonator respectively. The adequacy of the method, even in the case of impedance mismatch, is demonstrated by comparing these model simulations with experimentally obtained AF curves for both flow-through coaxial stub resonators and microstrip resonators immersed in the fluid under investigation.

1. Introduction

There are different ways to determine the dielectric permittivity of fluids. In dielectric spectroscopy [1–3], an alternating electric field is applied across two capacitor plates with the fluid under investigation as dielectric [4–7]. Measuring the impedance of the system as a function of frequency gives the dielectric permittivity, loss tangent, as well as their frequency dependence. Another known technique to assess the dielectric properties of a fluid is to apply a quarter wave length coaxial stub resonator as a sensing element by placing the fluid under investigation between its inner and outer conductor [2,3]. Since this method is based on the concept of a transmission line, it is less sensitive to errors caused by the parasitic capacitance and inductance of all elements in the measurement set-up as compared to the previously mentioned capacitance-based measurements, especially at high frequencies [8–10]. In a previous attempt we applied a lumped element model to a quarter wave length open-ended coaxial stub resonator [2,3]. Even though this first model was shown to adequately predict the amplitude versus frequency plot near the base resonant frequency of the stub resonator, it was limited to the fundamental (basic) resonant frequency. In the present contribution, we extend our first model for lossy stub resonator systems to a model that describes all resonance frequencies within a defined frequency range. The model, based on transmission line theory, comprises a general solution of the telegrapher's equations and takes into account both the skin effect and dielectric losses [11,12]. A particular challenge arises when the impedance of the stub resonator does not match the impedance of the transmission lines, the latter typically 50 Ω.

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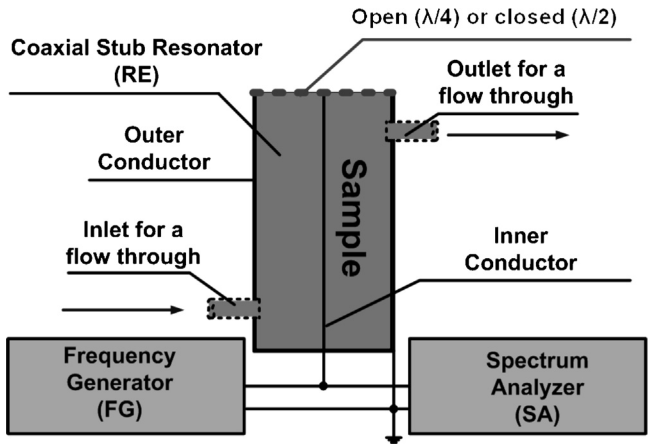

Fig. 1. Schematic outline of the coaxial stub resonator sensing system consisting of a function generator (FG), a spectrum analyzer (SA) and the coaxial stub resonator (RE). The dotted Inlet and Outlet indicate that the batch resonator can be optionally used as flow-through resonator. The liquid sample under investigation is applied as dielectric between inner and outer conductor of the coaxial stub resonator.

The feasibility of our method is demonstrated and discussed by comparing model simulations with experimentally obtained data using either a coaxial stub resonator or a microstrip line resonator. The results show that even if the system elements are unmatched, achieved by adapting the geometry of the resonator, the model still describes the experimental data quite well.

2. Model of the lossy stub resonator system

Fig. 1 gives a schematic overview of the coaxial stub resonator sensing system used in this study for analysing the dielectric properties of a fluid.

Fig. 2 shows the equivalent electrical circuit used of the experimental set-up in Fig. 1 comprising the frequency generator (FG) with internal impedance Z_s; transmission line TL1, connecting the function generator with the coaxial stub resonator, and transmission line TL2, connecting the coaxial stub resonator to spectrum analyzer (SA) with internal impedance Z_{SA}. The coaxial stub resonator T, including a connector, are represented as transmission line TL3 with characteristic impedance Z_{cRE} and described by a distributed element model.

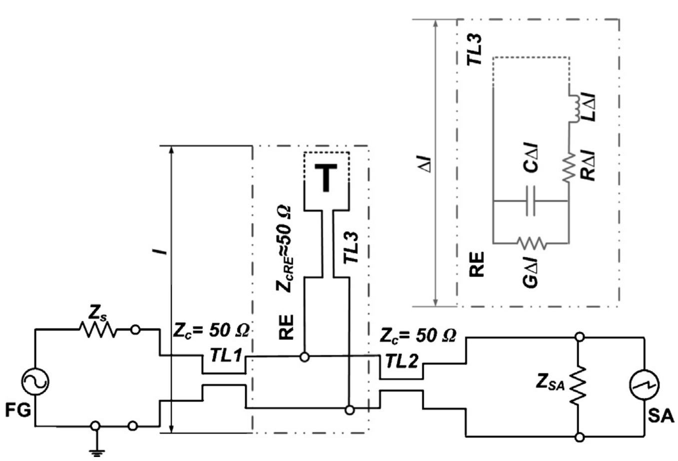

Fig. 2. The equivalent electrical circuit of the sensor system shown in Fig. 1. Parameters $L\Delta l$, $C\Delta l$, $G\Delta l$ and $R\Delta l$ represent the resonator's distributed element inductance, capacitance, conductivity and resistance, respectively, all with length Δl. The dotted line on T indicates either an open or closed (short) circuit, representing a λ/4 or λ/2 resonator, respectively.

Based on the equivalent electric circuit of the sensor system in Fig. 2, we will now derive a straightforward model for predicting the amplitude versus frequency (AF) plot of a coaxial sensing system. The model is straightforward in the sense that it is based on and by implication follows directly from the classic telegraph equations.

Given a resonator length l of λ/4, the distributed element inductance L, resistance R, capacitance C and conductance G of a coaxial transmission line are expressed by Eqs. (1)–(6), respectively [11,14]:

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right) + \frac{\mu_0\mu_r}{2\pi}\sqrt{\frac{\rho}{2\omega\mu_0\mu_r}}\left(\frac{1}{d}+\frac{1}{D}\right) \quad (1)$$

$$R = \frac{\rho}{2\pi\delta_s}\left(\frac{1}{0.5\cdot D}+\frac{1}{0.5\cdot d}\right) \quad (2)$$

$$\delta_s = \sqrt{\frac{2\rho}{\omega\mu_r\mu_0}} \quad (3)$$

$$R_s = \sqrt{\frac{\omega\mu_0\mu_r\rho}{2}} = \frac{\rho}{\delta_s} \quad (4)$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(D/d)} \quad (5)$$

$$G = \omega\cdot C\cdot \tan\ \delta \quad (6)$$

where μ_0, magnetic permeability of free space (vacuum permeability) [H m^{-1}]: $\mu_0 = 4\pi\times10^{-7}$; μ_r, relative magnetic permeability [–]; ω, angular frequency, $\omega = 2\pi f$ [rad/s]; ω_0, angular resonance frequency, $\omega_0 = 2\pi f_0$ [rad/s]; ε_0, dielectric permittivity of free space (vacuum permittivity) [F m^{-1}]: $\varepsilon_0 = 1/(\mu_0\cdot c)^{-1/2}$; c, speed of light in vacuum [m/s]; ε_r, relative dielectric permittivity, [–]; D, (inner) diameter of outer conductor [m]; d, (outer) diameter of inner conductor [m]; R_s, surface resistance of the metal [Ω]; δ_s, depth of penetration [m]; ρ, specific resistance of the metal [Ω m]: $\rho = 1/\sigma$; σ, conductivity [S/m]; $\tan\delta$, dielectric loss tangent [–].

The resistance R in Eq. (2) is related to two physical parameters, the depth of penetration δ_s in both inner and outer conductor (skin effect) and the surface resistance of the metal R_s [11,15].

It is noted that the inner and outer conductor should be made of the same material; otherwise the model should account for two different values of the specific resistance of the metal ρ in Eq. (1).

Apart from the geometry of the device (D and d), medium property μ_r and material characteristics ρ, Eqs. (1)–(6) contain two (unknown) parameters needed to calculate L, C and G, i.e., ε_r and $\tan\delta$. The loss tangent, $\tan\delta$, in Eq. (6) represents dielectric losses in the fluid sample under investigation and is, like ε_r, a key parameter characterizing dielectric fluid properties. For pure, single component fluids, values of these parameters may be found in the literature. If the resonator is filled with a fluid of unknown composition, both can be determined by fitting the model simulations developed in this study against experimentally determined AF plots with the real part of the complex dielectric permittivity ε_{re} and $\tan\delta$ as free fitting parameters.

In an ideal resonator without any losses (i.e. $R = G = 0$) or, in general, whenever $R/L = G/C$ [15], the resonance frequency f_{res} of an open ended (λ/4) and closed end (λ/2) resonator are given by Eqs. (7a) and (7b), respectively [2]. In these special cases, the dielectric constant ε_{re} can be determined directly from Eqs. (7a) and (7b) [11,12]:

$$f_{res} = (2n-1)/(2\cdot\pi\cdot\sqrt{LC}) = c\cdot(2n-1)/(4\cdot l\cdot\sqrt{\varepsilon_{re}\varepsilon_0\mu_r\mu_0}) \quad (7a)$$

$$f_{res} = n/(2\cdot\pi\cdot\sqrt{LC}) = c\cdot n/(2\cdot l\cdot\sqrt{\varepsilon_{re}\varepsilon_0\mu_r\mu_0}) \quad (7b)$$

where n, the order number of f_{res} [Hz]; l, the length of the resonator [m].

In all other cases with losses, the minimum in the AF response obtained with the set-up shown in Fig. 1 is not defined exclusively