Physica A 510 (2018) 552–557

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Magnitude and variation of the critical power law exponent and its physical controls

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HIGHLIGHTS

- Physical mechanism of variation of the critical power law precursor exponent.
- Range of the critical power law exponent.
- Relationship of critical exponent with the degree of local stress controlling damage.

ARTICLE INFO

Article history:
Received 17 March 2018
Received in revised form 23 June 2018
Available online xxxx

Keywords:
Critical power law precursor
Failure
Exponent
Magnitude

ABSTRACT

We study the physical controls on the scatter of exponents in the critical power law relation that describes an acceleration in precursory signals of deformation (displacements) or seismicity (damage) in the vicinity of failure time. Based on the time-dependent fiber bundle model and equal load share (ELS) rule, we find that the critical power law exponents range from −0.5 to −1.0. And values of the critical power law exponents depends on a parameter ρ, which defines the sensitivity of damage growth in a fiber to the local stress. Both the simulation results and theoretical analysis demonstrate that the critical power law precursor exponent $-\beta$ has a relationship $-\beta = -(1 - 1/\rho)$ with ρ. Thus, our results illustrate a physical mechanism of variation of the critical power law exponent that is determined by the degree of the local stress controlling the damage evolution of a fiber.

1. Introduction

Monitoring the critical acceleration of measurable quantities such as displacements or (micro-)seismicity is widely accepted as a proxy for a precursor for failure and for the prediction of time-to-failure [1–8]. This is often analyzed as a critical phenomenon [9–12] with a power-law divergence of physical quantities at failure time t_f. In the vicinity of the failure time, the critical acceleration exhibited by a measurable quantity can be described as a power law relation [1–3,7,8,13–15] with respect to the time to failure

$$\dot{\Omega} = k\left(t_f - t\right)^{-\beta}. \tag{1}$$

where Ω represents a measurable quantity and the overscripted dot represents the derivative of Ω with respect to time. $-\beta$ is the critical power law exponent. Eq. (1) can be deduced from the Voight relation [1,2,7,8,13]

$$\dot{\Omega}^{-\alpha}\ \ddot{\Omega} - A = 0 \tag{2}$$

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https://doi.org/10.1016/j.physa.2018.07.020

that describes the behavior of materials in their terminal stage of failure, with $k = [A(\alpha-1)]^{1/(1-\alpha)}$ and $\beta = 1/(\alpha-1)$[1]. Where A and α are constants recovered from curve-fitting of observations.

Eq. (1) can be rewritten in a linearized form [14]

$$\dot{\Omega}^{-1/\beta} = k^{-1/\beta}\left(t_f - t\right). \tag{3}$$

Then the failure time can be determined by linearly extrapolating the curve of $\dot{\Omega}^{-1/\beta}$ versus time to its intersection with the time axis. This application of Voight's relation is known as the Failure Forecast Method (FFM) [14–17]. This method has been demonstrated to work well in the retrospective prediction of laboratory experiments [18,19], landslides [20,21], and volcanic eruptions [2–5,22]. In application of the FFM, the linearization of the power law trend is usually used to predict the failure time. This leads to a biased and imprecise prediction compared to a fully nonlinear fitting method because the error structure is no longer Gaussian after the transformation of the rate data [23].

However, the uncertainty resulting from the scatter of the critical exponent $-\beta$ is a key difficulty in using such method [24]. For example, Voight [1] discovered that $-\beta$ is approximately -1.0 for soils. For creep-relaxation experiments on rock, Hao et al. [25] found that the exponent $-\beta$ is approximately $-2/3$. Smith and Kilburn [22] found that $-\beta$ took values of approximately -0.43 for 3 data points for the 1991 Mount Pinatubo eruption (Philippines) and tend to -0.91 for 7 data points. Analysis [8,26–28] based on fiber bundle models and global mean-field approximations demonstrated that $-\beta = -0.5$. It has also been observed that precursory signals may evolve with time from conditions for $\alpha \approx 1$ to conditions for $\alpha \approx 2 (-\beta = -1.0)$ [5,6,20]. Thus, it is particularly important to understand the underlying mechanisms for the magnitude and variation of critical power-law exponents.

In materials science and engineering, the fiber bundle model has been widely used [29–31] to analyze the failure properties of heterogeneous materials. A time-dependent implementation of this method was first proposed by Coleman [31], representing the response of fiber bundles where each fiber has an independent lifetime drawn from some distribution. According to the load sharing rules, the fiber bundle models can be divided into two extreme types: equal load sharing (ELS) and the local load sharing (LLS) models [32–34].

The local load-sharing (LLS) fiber bundle model was introduced by Harlow and Phoenix [32,33]. In this model, when a fiber fails, the load it carried is redistributed equally onto its two nearest surviving neighbors. Hence, a survival fiber carries the load

$$\sigma = K\ \sigma_0, \tag{4}$$

with the stress concentration factor

$$K = 1 + (l + r)/2. \tag{5}$$

where l and r represent the number of broken fibers on the left or right of this survival fiber respectively. σ_0 is the load on every fiber at the initial state when all fibers are intact.

In the ELS model, the load is shared equally by all surviving elements in the system. Thus if N_{b} fibers have broken, every surviving fiber has the same stress concentration factor

$$K = \frac{N}{N - N_b(t)} = \frac{1}{1 - N_b/N} \tag{6}$$

and carries the same stress $\sigma = K\ \sigma_0$. $N_b(t)$ is the number of broken fibers up to time t. ELS model has a merit to give a theoretical analysis of continuous cases because some closed form analytic results can be obtained.

In this paper, we perform an analytical derivation about the critical power law precursor based on the ELS rule. We illustrate that the critical power law exponents change from -0.5 to -1.0. The relationship between the critical exponent with the parameter that reflects the nonlinear relationship between damage rate of a fiber and the local stress, is derived. Then, Monte Carlo simulations are performed to explicitly show the magnitude and variation of the critical exponent.

2. Bundle geometry and stochastic fiber lifetime model

We consider a fiber bundle consisting of N fibers that are arranged around the circumference of a circle, with each of them with two adjacent neighbors. We apply a fixed load $N\sigma_0$ to the bundle; that is, initially each fiber is intact and carries load σ_0. As time progresses, fibers break and thus surviving fibers share load according to the desired load redistribution rule.

When a fiber is subjected to a load history $\sigma(t)$, $t > 0$, its consumed lifetime during this loading process is described by [31,34]

$$T(t) = \int_0^t \kappa(\sigma(s))\, ds. \tag{7}$$

where $\kappa(\sigma)$ is usually referred to as the breakdown rule [30,33], and is usually described by a power law relation [34–36]

$$\kappa(\sigma) = \kappa_0 \left(\frac{\sigma}{\eta}\right)^{\rho}. \tag{8}$$