Physica A 509 (2018) 1102–1109

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

On the simultaneous estimation of delay model parameters in economic dynamics☆

Alexander V. Prasolov

Saint Petersburg State University, Russian Federation

ARTICLE INFO

Article history:
Received 26 January 2018
Received in revised form 10 April 2018
Available online xxxx

Keywords:
Mathematical models
Time lag
Identification
"Predator–prey" model

ABSTRACT

Mathematical models in the form of differential equations with a time lag have been widely used in economics, biology, engineering and medicine to model dynamical interactions. In this paper, a heuristic estimation algorithm of delay values is offered in discrete deterministic systems by minimizing the average quadratic deviation for parameter identification. A well-known "predator–prey" model falls within the solution set we offer and is widely used in economics. Obviously, real data requires the analysis of random measurement innovations be taken into account; however, this aspect is not considered for the sake of convenience.

1. Introduction

Modelling of dynamic processes in different branches of science is based on the following generally accepted approaches: balance proportions, regression analysis, heuristic assumptions and others, as discussed below.

Models of the controlled rotation of a solid body

A heavy solid body is rotating around its mass centre; the equation is written in the vector form with the coordinate system fixed within the body as

$$\theta\dot{\omega} + \omega \times \theta\omega = M\left(t-h\right),$$

where Θ - tensor of inertia (responsible for mass distribution in the solid body), ω - vector of angle rotation speed, and $M(t-h)$ - moment of external forces in the previous time period (i.e., delay in time on h units) [1].

Population volume of a biological species in the "predator–prey" task

$$\begin{cases} \dot{N}_1\left(t\right) = N_1(t)\left[-\varepsilon_1 - \gamma_{11}N_1\left(t-h\right) + \gamma_{12}N_2\left(t-h\right)\right] \\ \dot{N}_2\left(t\right) = N_2(t)\left[\varepsilon_2 - \gamma_{21}N_1\left(t-h\right) + \gamma_{22}N_2\left(t-h\right)\right]. \end{cases}$$

Here, $N_1(t)$ and $N_2(t)$ - volumes of the predator and prey populations, respectively, and h - the period of reproduction [2].

Model of economic growth with investments

$$\dot{K}\left(t\right) = -\mu K\left(t\right) + sF\left(K\left(t-h\right), L\left(t-h\right)\right),$$

where K - basic capital; μ, s - coefficients of amortization and saving, respectively; F - production function; and h - time of investment realization [3].

☆ This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.
E-mail address: a.prasolov@spbu.ru.

https://doi.org/10.1016/j.physa.2018.06.104
0378-4371/

Similarly, a delay appears in the bank reaction to changes in demand and supply in the credit-deposit process or in the mathematical description of the process of microorganism resistance to antibiotics.

In all examples given above, one cannot say how exactly to equate a delay. Thus, non-linear differential equations with lag need algorithms for lag estimation.

However, the building of a mathematical model in the form of balance proportions with continuous time creates difficulties in parameter identification of various models. This occurs because observations of the value of model variables are conducted in discrete time, i.e., with some time step. The transfer from continuous time to discrete time and back is connected to the resulting ambiguity, which can be enumerable as well as non-enumerable. Sometimes the use of a mathematical model with discrete time is enough for modelling purposes (forecasting, management). In such cases, both observations and modelling results are given in discrete time. This is exactly what occurs in the majority of economic tasks. Therefore, it is possible not to consider differential equations but only difference equations. Thus, taken as a mathematical model, we consider the autoregression of order p:

$$x_i(k) = \sum_{m=1}^{p} \sum_{j=1}^{n} a_{imj} x_j(k-m).$$

This proportion is linear with its parameters a_{imj}, so it is easy to apply the least squares method to find the best parameter values.

Moreover, if we consider that the measurement of current variables includes errors, it is then appropriate to introduce statistical methods for parameter estimation. One can single out statistically important parameters and test statistical hypotheses.

The object under consideration in this paper is the determination of a delay value, such as the determination of autoregression order in the case of autoregression. In mathematical economics, the following terminology is accepted: time shift on one unit is called lagging or shift on one lag. Thus, the above formula of autoregression contains p lags. There are no analytical methods to obtain the best p due to non-linearity on p of the autoregression formula as well as because this formula is more common, i.e., it represents a distributed lag. The task of building models with a distributed lag is excluded from the scope of this work. The standard approach is to separate (either by values or statistics) the most important additive components in autoregression and to cut less important values for lag localization additive components. Well-known tests [4–7] are devoted to these approaches.

The offered method has a heuristic character, but it is universal as it gives a result in any case. For instance, it does not matter whether the kind of a model is known or not, if there are random errors in the observation or there are no errors, if continuous time is used or not. The result, which is the estimation of the delay value, is chosen from the finite number of minimums of some function. If observed data are exactly relevant to their change laws, then the result will be exact. Otherwise, estimation errors will always exist. The novelty of the offered argument is relative, as there exist many algorithms to determine the order of autoregression, but we set up the delay value estimation problem for differential equations with continuous time. This requires the choice of a special fixed lag. The solution of the problem by a standard method would lead to obtaining a distributed lag. Apart from this, our method can be transformed for applications in many non-linear systems with delay.

In the following paragraphs, the method is demonstrated on a linear system and on the Lotka–Volterra model of the second order, which is used in economic tasks to describe the competitive interaction of economic agents.

2. Linear system identification

First, let us consider an elementary equation with a lag:

$$\dot{x} = -x(t-h),$$

where $h>0$ is a lag. Let us observe its solution $x(t)$ at $t = k\delta$, where $\delta>0$ is a step of observation discreteness, and $k = 0, 1, 2 \ldots, N$. Assume also that $h \gg \delta$ Write the equation in the integral form:

$$x(t) = x(h) - \int_0^{t-h} x(s)\,ds, \quad t \in [h, N\delta]. \tag{1}$$

Approximately replacing the integral by a sum and the delay h by an integer l of the discrete step δ, we calculate the below equation based on (1):

$$x(k\delta) = x(l\delta) - \delta \sum_{i=0}^{k-l-1} x(i\delta). \tag{2}$$

Let us introduce the functional of square deviations:

$$F(l) = \sum_{k=l+1}^{N} \left(x_k - x_l + \delta \sum_{i=0}^{k-l-1} x_i \right)^2 \frac{1}{N-l}. \tag{3}$$