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The random field Ising model in a shifted bimodal probability distribution

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HIGHLIGHTS

- Ising model in the presence of an external random magnetic field.
- The external random magnetic field follows the shifted bimodal probability function.
- The combined system possesses first and second order phases transitions joint at a tricritical point.
- The occupation probabilities depend on the random magnetic field strength.

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ABSTRACT

The critical behavior of the Ising model in the presence of a random magnetic field is investigated for any temperature T. The random field is drawn from the proposed shifted bimodal probability distribution $P(h_i) = \left(\frac{1}{2} + \frac{1}{2h_0}\right)h_i\delta(h_i - h_0) + \left(\frac{1}{2} - \frac{1}{2h_0}\right)h_i\delta(h_i + h_0)$, h_i is the random field variable with strength h_0. By obtaining data for several transition temperatures T and random field strengths h_0, we conclude that the system possesses first and second order phase transitions, joined smoothly at a tricritical point, with coordinates $(T^{TCP}, h_0^{TCP}, V_0^{TCP}) = (1.5775571, 3.7348565, -4.7741775)$, where V_0 is an auxiliary field. Using the variational principle, we determine the phase diagram and the equilibrium equation for magnetization (with zero and nonzero values), solve it for both transitions and at the tricritical point and examine the stability conditions of each phase transition.

1. Introduction

One of the most extensively studied models in Statistical Physics is the Ising model; this was examined not only in its pure form but in various other versions by modifying it appropriately for comparing reliably the theoretical predictions with the experimental results; one such version is the one in the presence of an external random magnetic field (ERMF) acting on its constituent particles (spins), consequently the combined system is called random field Ising model (RFIM) [1–5]. The concept of the RFIM is closely connected to the choice of a suitable probability density function (PDF).

A significant part of the study is the chosen PDF for the ERME; as such ones are the discrete symmetric bimodal with equal probability $p = q = \frac{1}{2}$ and the continuous Gaussian. In the last few years, the asymmetric bimodal PDF ($p \neq q$) was proposed, as well as the anisotropic (unequal random fields in the directions $\pm z$); in addition to the bimodal, the trimodal (asymmetric-anisotropic) was also used [6–20]. The general form of the asymmetric bimodal PDF is

$$P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + h_0) \tag{1}$$

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where p is the fraction of lattice sites interacting with the magnetic field h_0, disorder strength, while the rest sites (of fraction q) interact with the inverse field $(-h_0)$ such that $p + q = 1$.

The RFIM Hamiltonian is,

$$H = -J \sum_{\langle i,j \rangle} S_i S_j - \sum_i h_i S_i, \qquad S_i = \pm 1 \tag{2}$$

The summation in the first term extends over all nearest neighbors and is denoted by $\langle i, j \rangle$; in the second term h_i represents an independent random field that couples to the one dimensional spin variable S_i. We consider that $J > 0$ so that the ground state is ferromagnetic in the absence of random fields.

In spite of the fact that much theoretical and experimental efforts have been invested for understanding RFIM, several aspects of it remain still unclear; the only well-established conclusion drawn was the existence of a phase transition for $d \geq 3$ at low temperature and weak disorder, that is, the critical lower dimension d_l is 2, while many other issues are still unanswered; among them is the order of the phase transition (first or second order), the universality class [3,21–24]. The important issue is the influence of the chosen PDF as it can yield a first order phase transition (FOPT) or a second order phase transition (SOPT) or both joined smoothly at a tricritical point (TCP) [6,18,25–37]. This discrepancy whether the phase transition is FOPT or SOPT relies on the relatively small value for the exponent β.

The experimental realization of random fields was the main issue. However, Fishman and Aharony [38] showed that the randomly quenched exchange interactions Ising antiferromagnet in a uniform field H is equivalent to a ferromagnet in a random field with the strength of the random field linearly proportional to the induced magnetization.

Because of the aforementioned significant uncertainties with the choice of the PDF, we propose a different version of the PDF in (1), which is called shifted bimodal PDF (as far as we are aware such a PDF has never been used earlier),

$$P(h_i) = p h_i \delta(h_i - h_0) + q h_i \delta(h_i + h_0) \tag{3}$$

with $p + q = 1$. A similar continuous PDF is the Rayleigh one,

$$P(x, \sigma) = \frac{x}{\sigma^2} \mathrm{e}^{-x^2/(2\sigma^2)} \tag{4}$$

with $x \geq 0$, used extensively in magnetic resonance imaging, communications theory, modeling wind speed, wave heights, sound/light radiation, in engineering to measure the lifetime of an object, although the putative discrete Rayleigh PDF is different [39].

From the normalization condition for $P(h_i)$ in (3) one gets the result $p - q = \frac{1}{h_0}$, which when combined with $p + q = 1$, yields

$$p = \left(\frac{1}{2} + \frac{1}{2h_0}\right), \quad q = \left(\frac{1}{2} - \frac{1}{2h_0}\right) \tag{5}$$

so that the probabilities p and q depend on the strength of the ERMF; p and q lie in the interval $[0, 1]$ consequently $h_0 \geq 1$ and the respective PDF (3) assumes the form

$$P(h_i) = \left(\frac{1}{2} + \frac{1}{2h_0}\right) h_i \delta(h_i - h_0) + \left(\frac{1}{2} - \frac{1}{2h_0}\right) h_i \delta(h_i + h_0) \tag{6}$$

Form the functional form of the PDF in (6) we conclude that for large values of h_0 this PDF converts into the symmetric shifted bimodal PDF ($p = q = \frac{1}{2}$); for $h_0 = 1$ only the first term survives with $p = 1$, $q = 0$.

In this investigation, we study the RFIM by considering as PDF the proposed one in the expression (6) in order to study the critical behavior with respect to T by varying the strength h_0. The paper is organized as follows: In the next section, the respective free energy and equation of state for the magnetization are derived for the Hamiltonian (2). In Section 3, the phase diagram, tricritical points and magnetization profiles for various T's are calculated; we close with the conclusions in Section 4.

2. The model

According to the MFA the Hamiltonian (2) of the RFIM takes on the form [6,7,18],

$$H_{MFA} = \frac{1}{2} N z J M^2 - \sum_i (zJM + h_i) S_i \tag{7}$$

where N is the number of spins, z the coordination number and M the magnetization; the respective free energy per spin within the MFA is,

$$\begin{aligned} \frac{1}{N} \langle F \rangle_h &= \frac{1}{2} z J M^2 - \frac{1}{\beta} \langle \ln\{2 \cosh[\beta(zJM + h_i)]\} \rangle_h \\ &= \frac{1}{2} z J M^2 - \frac{1}{\beta} \int P(h_i) \ln\{2 \cosh[\beta(zJM + h_i)]\} dh_i \end{aligned} \tag{8}$$