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Group formation in the spatial public goods game with continuous strategies

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HIGHLIGHTS

- We study group formation in the public goods game with continuous strategies.
- A mechanism of failure and recovery is proposed.
- An active group becomes inactive with the probability depending on the contribution.
- An inactive group recovers to be active with some probability.

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ABSTRACT

We study group formation in the spatial public goods game with continuous strategies. An active group will become inactive with high (low) probability if the collective contribution is below (above) a threshold value. Meanwhile, an inactive group recovers to be active with some probability. We have found that the cooperation level and the average payoff of players are maximized at moderate values of the threshold and the recovery rate. Spatial distributions of strategies and active groups are plotted to understand the evolution of cooperation.

1. Introduction

Understanding the persistence of cooperation among selfish individuals remains a challenge. So far, evolutionary game theory has provided a powerful mathematical framework to address this problem. Researchers have proposed various game models, among which the public goods game (PGG) has been a prevailing paradigm [1]. The classical PGG involves an interacting group with N players. Each player may choose either of two strategies: cooperation or defection. Cooperators contribute to the group with a cost, whereas defectors do not contribute. The collective contribution is multiplied by an enhancement factor and is then divided equally among all group members, regardless of whether they contribute or not. Due to the rapid development of complex networks, the PGG and other evolutionary game models have been extensively studied in various kinds of structured populations [2–5], including regular lattices [6–12], small-world networks [13,14], scale-free networks [15,16] and interdependent networks [17–19]. It has been found that punishment [20,21] and heterogeneity [22,23] play important roles in the spatial PGG.

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In the spatial PGG, each group is composed of a focal player and all its nearest neighbors. For a given network, the number of different PGG groups is equal to the network size. In most of the previous studies, each PGG group is assumed to be existent all the time. However, in real life, not all groups can be successfully organized and some groups would be closed for various reasons. For example, many companies go bankrupt during the period of economic recession. Very recently, Szolnoki and Chen proposed a model where only those players whose payoff exceeds a threshold level can establish a PGG group in the next time step [24]. They found that a carefully chosen threshold to establish PGG group could efficiently improve the cooperation level.

In most of the previous studies, the contribution of each cooperator is restricted to a certain value. However, diverse contributions are ubiquitous in the real world. For example, to prevent global warming, the reduction in the emission of greenhouse gas from distinct countries could be different. Another example is that people invest varying amount of money in the stock market. Therefore, the continuous version of the public goods games (CPGG) can be considered an additional step toward more realistic condition. Wang *et al*. demonstrated that increasing the diversity of contributions greatly reduces the probability of finding the population in a homogeneous state full of defectors [25]. Chen *et al*. studied impact of generalized benefit functions on the evolution of cooperation in the CPGG [26].

In this paper, we study group formation in the CPGG. Note that the benefits of group members are positively related with the collected contribution. According to the assumption of economic man, group members probably quit the group if the collected contribution is too low, leading to the dissolution of a group [27]. On the other hand, the maintain of a group is usually costly in real life. A company cannot be established if it does not receive enough investment. Based on the above considerations, we propose a mechanism of group formation driven by the collected contribution. Motivated by the study of failure and recovery in complex networks [28,29], we assume that an active (open) group will become inactive (closed) with higher probability when the collective contribution of the group is less than a threshold, which can be regarded as the operating cost for a group. On the other hand, an inactive group will recover to be active with some probability. We have found that there exists optimal values of the threshold and the recovery rate, leading to the highest cooperation level.

2. Model

Players are located on a $L \times L$ square lattice with periodic boundary conditions. Each active CPGG group is composed of a focal player (so-called leader) and its four neighbors. If all groups are active, a player x participates in five different CPGG groups organized by x and its four neighbors respectively. Initially, the strategy of each player x is drawn uniformly at random from the unit interval $s_x \in [0, 1]$, defining its level of contribution in each of the involved and active groups. While the limits 0 and 1 recover the pure defection and pure cooperation, intermediate values from the unit interval correspond to more or less cooperative players.

The collective contribution of an active group organized by player i at time t is

$$B_i(t) = \sum_{x=0}^{4} s_x(t), \tag{1}$$

where $x = 0$ stands for player i and $x > 0$ represent the neighbors of i. The collective contribution is multiplied by a factor r, and is then redistributed uniformly to all the five players in this group. At time t, the payoff that player j gains from an active group organized by player i is

$$\Pi_j^i(t) = -s_j(t) + \frac{rB_i(t)}{5}. \tag{2}$$

The total payoff of the player j is calculated by

$$P_j(t) = \sum_{i \in \Omega_j} \Delta_i(t) \Pi_j^i(t), \tag{3}$$

where Ω_j denotes the community of neighbors of j and itself, $\Delta_i(t) = 1$ if the group organized by player i at time t is active otherwise $\Delta_i(t) = 0$.

Initially ($t = 0$) all groups are assumed to be active. An active group becomes inactive with the probability:

$$W_1[\Delta_i(t) = 1 \to \Delta_i(t+1) = 0] = \frac{1}{1 + e^{[B_i(t) - H]/K}}, \tag{4}$$

where H represents the threshold value and K determines the level of uncertainty in the dissolution of groups. An active group is more likely to become inactive for the higher value of H. Meanwhile, an inactive group recovers to be active with the probability:

$$W_2[\Delta_i(t) = 0 \to \Delta_i(t+1) = 1] = \mu, \tag{5}$$

where the recovery rate $0 \le \mu \le 1$. For $\mu = 0$, a group cannot be reorganized once it is closed down. For $\mu = 1$, an inactive group is determinately recovered in the next time step.