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Analysis of mixed traffic flow with human-driving and autonomous cars based on car-following model

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HIGHLIGHTS

- A mixed traffic flow was investigated based on car-following model.
- One theorem were proven as criteria for judging stability of traffic flow.
- Fundamental diagrams were examined with different parameters in simulations.

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ABSTRACT

We investigated the mixed traffic flow with human-driving and autonomous cars. A new mathematical model with adjustable sensitivity and smooth factor was proposed to describe the autonomous car's moving behavior in which smooth factor is used to balance the front and back headway in a flow. A lemma and a theorem were proved to support the stability criteria in traffic flow. A series of simulations were carried out to analyze the mixed traffic flow. The fundamental diagrams were obtained from the numerical simulation results. The varying sensitivity and smooth factor of autonomous cars affect traffic flux, which exhibits opposite varying tendency with increasing parameters before and after the critical density. Moreover, the sensitivity of sensors and smooth factors play an important role in stabilizing the mixed traffic flow and suppressing the traffic jam.

1. Introduction

Autonomous car has been a hot issue for a couple of decades and so will in the future. It is a fact that human-driving cars and autonomous cars will appear at the same time in urban city. How will these two types of cars operate together? Modeling on mixed traffic flow is a general tool to solve this problem. Car-following model as one of microscopic traffic flow models has been developed for many decades and was used widely to describe the moving behavior of individual car in a system. Pipes [1] firstly proposed the classical car-following model to describe a stream of cars' behavior in 1953. After that, so many improved car-following models were proposed to describe the traffic dynamics more and more realistically. Bando et al. [2] improved the Pipes' car-following model by substituting the front car's velocity with the optimal velocity function and overcame the shortcoming of the classical models. Holland [3] developed a general criterion to determine whether a catastrophic event will occur. The general criterion was in accordance with the stability condition of the car-following models. Nagatani [4] and Sawada [5] improved the optimal velocity model by considering the next-nearest-neighbor interaction. The stabilization of the traffic flow was greatly enhanced and the solution of the mKdV equation was obtained. Jiang et al. [6] built the

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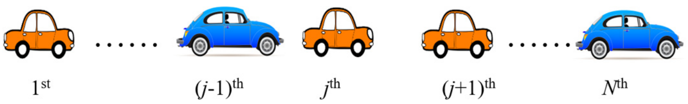

Fig. 1. The illustration of the N-car system with human-driving cars (blue) and autonomous cars (orange). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

full velocity difference model based on an improved optimal velocity model. The relative velocity between the current car and the immediately one has a prominent effect on the stability of the traffic. Lenz et al. [7] and Nakayama et al. [8,9] extended optimal velocity model with forward-looking and backward-looking effects in a cooperative driving system. The stability of uniform flow is strengthened. Zhang et al. [10] proposed a new car-following model to describe the multiphase traffic. The so-called capacity drop and traffic hysteresis loop was reproduced. Ge et al. [11] and Li et al. [12] analyzed the extended optimal velocity model in an intelligent transportation system environment with a nonlinear method. Yu et al. [13] investigated the kink–antikink properties of an extended optimal velocity model. Peng [14] analyzed a new lattice model with the consideration of individual difference of anticipation driving behavior based on car-following theory. Yu et al. [15] improved the optimal velocity model with a consideration of not only the full velocity difference but also the acceleration difference. Ngoduy [16] studied the instability of the traffic flow analytically. An application of intelligent vehicle was a potential solution to eliminate the instability of the traffic flow. The stability threshold of heterogeneous traffic flow was found with a linear analysis method. Tang et al. [17,18] proposed new extended version car-following model by considering the communication of inter-vehicle and its liability. Zhang et al. [19] investigated the mixed traffic flow with an extended macroscopic kinetic wave traffic flow model. Two types of vehicles (passenger car and truck) were considered with identical free flow speed. This model can be used to study long crowded highway. Ge et al. [20] proposed acceleration-based connected cruise control to increase roadway traffic mobility. Acceleration-based connected cruise control is designed to be able to use acceleration signals received from multiple vehicles ahead through wireless vehicle-to-vehicle (V2V) communication. This provides robust performance and remains scalable for large systems of connected vehicles. Ge et al. [21] and Redhu et al. [22] respectively applied the delay-feedback control scheme to improve the traffic flow behavior. These methods were proved valid in suppressing the traffic jam. Yu and Shi [23–25] proposed two new connected cruise control strategies considering multiple preceding cars' velocity changes with memory and vehicular gap fluctuation respectively to improve roadway traffic mobility, to enhance traffic safety and to reduce fuel consumptions and exhaust emission. They verify through the numerical simulation that the models are valid. Work et al. [26] investigated traffic stream with mixed human operated and automated vehicles. Based on previous models a second-order-continuum traffic flow model was built to describe the mixed traffic stream. The numerical results verified that it is valid. Tang et al. [27,28] proposed an extended macro traffic flow model accounting for the driver's bounded rationality and a speed guidance model accounting for the driver's bounded rationality at a signalized intersection. The driver's bounded rationality plays important role in influencing the behavior of traffic flow. Moreover, Tang et al. [29,30] investigated the impacts of energy consumption and emissions on the trip cost without late arrival at the equilibrium state and proposed an extended car-following model to study the influences of the driver's bounded rationality on his/her micro driving behavior, and the fuel consumption. Recently, Tang et al. [31] proposed a car-following model to investigate the impacts of signal light on driving behavior, fuel consumption and emissions during the whole process that each vehicle runs across the intersection. They are interesting. These above literatures mainly discussed some car-following models related to cooperative driving, intelligent transportation system and control mechanism so on. The mixed traffic flow with human driving cars and autonomous cars were few involved. In this paper, we will study the fundamental diagrams and density waves of the mixed traffic flow by using car-following model.

The remainders of this paper are organized as follows. In Section 2, the dynamical models for human-driving car and autonomous car were presented. A lemma and a theorem were given successively. In Section 3, numerical simulations were carried out in three situations. Some new results were exhibited with figures. In Section 4, the summary is given.

2. Model

Assume that a stream of traffic flow mixed with human-driving and autonomous cars move forward on a single lane highway without any overtaking under periodical boundary condition. The illustration sketch is plotted in Fig. 1 that N cars distribute on the road homogeneously (human driving cars are blue and autonomous cars are orange.) Under periodical boundary condition the Nth car is the leading one and the $(N+1)$th car is the first one. It is no doubt that the driving modes of these two types of cars are different. Therefore, the motion equations are not the same. Let us discuss the dynamical model of human-driving car and autonomous car respectively.

2.1. Dynamical model for human-driving cars

As well known that Bando's model is suitable for describing the human driving cars' behavior. It is formulated as

$$\ddot{x}_j(t) = a \times [V(\Delta x_j(t)) - \dot{x}_j(t)] \tag{1}$$