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Pricing geometric Asian rainbow options under fractional Brownian motion

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HIGHLIGHTS

- A pricing model for Asian rainbow options under fractional Brownian motions is provided.
- Closed form for the price of the geometric Asian rainbow option is derived for the first time.
- Simulation experiments and sensitivity analyses show that the proposed model is a reasonable one.

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ABSTRACT

In this paper, we explore the pricing of the assets of Asian rainbow options under the condition that the assets have self-similar and long-range dependence characteristics. Based on the principle of no arbitrage, stochastic differential equation, and partial differential equation, we obtain the pricing formula for two-asset rainbow options under fractional Brownian motion. Next, our Monte Carlo simulation experiments show that the derived pricing formula is accurate and effective. Finally, our sensitivity analysis of the influence of important parameters, such as the risk-free rate, Hurst exponent, and correlation coefficient, on the prices of Asian rainbow options further illustrate the rationality of our pricing model.

1. Introduction

With constant change in financial markets, the derivative markets have become increasingly risky in recent years. Derivatives, in particular, face potentially large losses during periods of market volatility. The collapse of Lehman Brothers in 2007 highlighted the risk associated with these derivatives. Meanwhile, reasonable pricing of complex derivatives in a manner that optimizes the market allocation can enhance market activity and control or reduce the level of financial market risk. Therefore, the accurate pricing of derivatives has been an ongoing problem in mathematical finance research. Rainbow options, which were first proposed by Margrabe [1], are an important type of "exotic option". These multifactor options involve the behavior of two or more underlying assets, and the aim is to select the best and worst multiple assets to buy or sell. As a result, these derivative products are more flexible than other options, especially in relation to international diversification under global asset allocation strategies. Stulz [2] provides a formula for the European-style rainbow option pricing of two assets. Johnson [3] extends the two-asset rainbow option pricing formula and finds a closed form formula. Rubinstein [4] deduces a rainbow option pricing formula under the risk-neutral hypothesis, which depends on the minimum or maximum price of the underlying asset. Hucki and Kolokoltsov [5] use game theory to investigate

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the problem of rainbow option pricing with fixed transaction costs. Meng et al. [6] develop an efficient pricing method for rainbow options based on sine–sine series expansions. Dockendorf et al. [7] study the European-style rainbow option pricing problem using stochastic and cointegration models.

Rainbow options typically vary from the traditional American and European options in that they are excellent tools for hedging risks that arise from holding multiple assets. Rainbow options are most commonly used when valuing natural resources, since they depend on both the price and quantity of a natural resource deposit. Furthermore, Asian options, a type of path-dependent option, are more robust in hedging and mitigating risk, and are the preferred approach for measuring market risk. Thus, the Asian rainbow options are developed whose payoff depends on the maximum or minimum of average prices of the underlying assets. However, a number of studies have shown that asset prices display self-similarity, long-range dependence, and volatility [8–10]. As fractional Brownian motion manifests the properties of self-similarity and long-range dependence, it has the ability to capture the typical behavior of financial returns. Accordingly, scholars are increasingly focusing on fractional Brownian motion as a basis for the option pricing problem [11–13]. As Rogers [14] states, "The characteristics of self-similarity and long-range correlation of fractional Brownian motion make it more suitable for describing the fluctuation of financial assets than geometric Brownian motion".

Although fractional Brownian motion is neither a Markov process nor a semimartingale, we cannot analyze it using the common stochastic calculus. However, Hu and Øksendal [15] point out that the Wick integral or fractional Itô integral method can be used to solve the aforementioned problem. Accordingly, the Black–Scholes formula under fractional Brownian motion can satisfy the basic assumptions, including no arbitrage and market completeness, in traditional pricing model. Based on this setting, we explore the problem of pricing Asian rainbow options in the fractional Brownian environment. Using the multidimensional fraction Itô integral and the principle of non-arbitrage, we deduce the pricing formula for two-asset rainbow option pricing under fractional Brownian motion. We further compare the difference between the analytical solution and Monte Carlo simulation solutions to confirm the accuracy of the Asian rainbow option pricing formula. Finally, a sensitivity analysis is conducted on the influence of important parameters, such as the risk-free interest rate, Hurst index, and asset correlation coefficient, on Asian-style rainbow option prices.

The rest of the paper is organized as follows. In Section 2, we present an analytic pricing formula for the geometric Asian rainbow options. In Section 3, we derive the simulation solutions of two-asset geometric Asian rainbow options. Section 4 contains sensitivity analysis of the parameters in the pricing formula. Concluding remarks are given in the last section.

2. Rainbow options pricing model under fractional Brownian motion

A generalization of classical Brownian motion, fractional Brownian motion is a centered Gaussian process with stationary increments [16]. However, the increments of fractional Brownian motion are not independent. When the Hurst index is greater than 1/2, fractional Brownian motion has positively correlated increments, which means historical fluctuations can have a lasting effect. Therefore, fractional Brownian motion is more suitable for describing the fluctuations of financial assets.

2.1. Illustrations

Let (Ω, F, F_t, P) be a probability space with σ-flow. F_t is a natural σ-algebra stream generated by fractional Brownian motion. P is a risk-neutral measure. $B_H = \{B_H(t)\}_{t\geq 0}$ is the fractional Brownian motion of (Ω, F, P), satisfying $B_0^H = E\left[B_t^H\right] = 0$, $E\left(B_t^H B_s^H\right) = \frac{1}{2}\left\{t^{2H} + s^{2H} - |t-s|^{2H}\right\}$. When Hurst parameter $H = 0.5$, B_t^H is often referred to as standard Brownian motion. Consider the process of the underlying asset price satisfying fractional Brownian motion, that is, $\{S_t : t \geq 0\}$ is the dynamic process governing the asset prices at time t, which follows the following distribution in the risk neutral measure P,

$$dS_t = \mu S_t dt + S_t \sigma dB_t^H, 0 \leq t \leq T, S_0 = S, \tag{1}$$

where S_t is the underlying asset price, μ is the expected rate of return on the underlying asset, σ is the asset price volatility, and B_t^H is the fractional Brownian motion of the Hurst index with $H \in \left(\frac{1}{2}, 1\right)$.

To derive the currency option pricing formula, we assume that the following assumptions hold:

(1) the dynamics of the underlying asset price follows fractional Brownian motion (fBM) with Hurst index $H > 1/2$;
(2) there are no transaction costs or taxes and all securities are perfectly divisible;
(3) security trading is continuous;
(4) the risk-free interest rate $r(t)$ is known and constant through time; and
(5) there are no riskless arbitrage opportunities.

2.2. Pricing formula of geometric Asian rainbow options

Under the assumption of continuous time, two special classes of asset, S_1 and S_2, have to meet the following differential equation:

$$\begin{cases} \dfrac{dS_{1t}}{S_{1t}} = \mu_1 dt + \sigma_1 dB_{1t}^H \\ \dfrac{dS_{2t}}{S_{2t}} = \mu_2 dt + \sigma_2 dB_{2t}^H \end{cases} \tag{2}$$