Physica A 495 (2018) 114–125

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Impact of rough potentials in rocked ratchet performance

S. Camargo [a,b,*], C. Anteneodo [c,d]

[a] Federal University of ABC, Santo André, Brazil
[b] EMAp, Fundação Getúlio Vargas, Rio de Janeiro, Brazil
[c] Department of Physics, PUC-Rio, Rio de Janeiro, Brazil
[d] National Institute of Science and Technology for Complex Systems, Brazil

HIGHLIGHTS

- We investigate the effect of roughness of the periodic potential in thermal ratchets.
- Flux and efficiency are calculated as a function of roughness parameters.
- Transport effects which cannot occur in smooth potentials are detected.
- Depending on the parameter values, roughness can enhance the performance.
- Our finding can have practical implications for microfabricated rough potentials.

ARTICLE INFO

Article history:
Received 21 July 2017
Received in revised form 11 December 2017
Available online 20 December 2017

Keywords:
Brownian ratchet
Transport processes
Roughness

ABSTRACT

We consider thermal ratchets modeled by overdamped Brownian motion in a spatially periodic potential with a tilting process, both unbiased on average. We investigate the impact of the introduction of roughness in the potential profile, over the flux and efficiency of the ratchet. Both amplitude and wavelength that characterize roughness are varied. We show that depending on the ratchet parameters, rugosity can either spoil or enhance the ratchet performance.

1. Introduction

Some mechanisms working in the Brownian domain are not as intuitive as they may appear. One of them is ratcheting, where periodic forces which are null on average can still produce directed motion [1]. The study of this effect had initially biophysical motivations, like the operation of molecular motors [2,3], and more recently mainly technological ones, such as in the microfabrication of devices that can be used to separate or rectify the motion of microparticles [4,5]. These implementations include, amongst other ones, silicon membranes [6], optical lattices [7–9], quantum dot arrays [10], and vortex rectifiers in superconducting films [11–13].

There are mainly three types of ratchets: (i) pulsating ratchets — the potential is switched on and off or there is a traveling potential, changing the barrier height; (ii) tilting ratchets — with the addition of fluctuating forces or a rocking periodic force, and (iii) temperature ratchets [1]. In our study, we choose a ratchet of the rocked type. Nonequilibrium fluctuations are introduced as an additive tilting force, including the periodic rocking plus noise, all unbiased on average. Following the definitions of symmetry and supersymmetry [1], the net flux vanishes for any combination of potential and tilting which are both symmetric or both supersymmetric. Otherwise a net flux arises.

* Corresponding author at: Federal University of ABC, Santo André, Brazil.
E-mail addresses: sabrina.camargo@ufabc.edu.br (S. Camargo), celia.fis@puc-rio.br (C. Anteneodo).

https://doi.org/10.1016/j.physa.2017.12.074
0378-4371/

While the effect of spatiotemporal asymmetries, nonequilibrium fluctuations and other features have been extensively studied [1,3,14–21], the role of roughness in the periodic potential has been less explored. However, spatial inhomogeneities or impurities can yield deviations from a smooth ratchet profile, with important implications in transition rates. In fact, the roughness of the potential is crucial in biophysical contexts such as in protein folding [22,23], where the potential surface can exhibit a rich structure of maxima and minima, hierarchical or not. The roughness in energy landscapes has been experimentally measured in proteins [24–27] but can also be microfabricated, for instance, holographic optical tweezers, used in optical ratchets, can generate complicated potential energy landscapes for Brownian particles [28].

Within this scenario, Zwanzig [29] studied diffusion in a rough potential, pointing to the reduction of the effective diffusion coefficient when compared to a smooth surface. Later, Marchesoni [30] explored disorder in a ratchet potential, due to impurities or randomness, reporting quenching of the effectiveness of thermal ratchets. More recently, Mondal et al. [31] showed that roughness hinders current significantly. In our work, we investigate the effects of perturbations of short wavelength superimposed on the periodic potential. Varying the amplitude and wavelength of these perturbations, we monitor the net directed current, as well as the efficiency, for a wide range of intensities of the time varying forces. We show that perturbations do not always spoil but, depending on the ratchet parameters, can enhance the performance.

The paper is organized as follows. We define the details of the ratchets and rugous perturbations of the potential in Section 2, and the methods in Section 3. Results for the effects of sinusoidal perturbations, over symmetric and asymmetric forms of the spatial periodic potential, in the adiabatic limit, are presented in Sections 4.1–4.3 and also in the Appendices. Other types of perturbations are analyzed in Section 4.4. The non-adiabatic regime is considered in Section 5. Concluding remarks are presented in Section 6.

2. The system

We consider the overdamped regime for a particle of unit mass obeying the equation of motion

$$\dot{x} = -U'(x) + F(t) + \zeta(t)\,, \tag{1}$$

where $U(x)$ is a spatially periodic potential, $F(t)$ a time periodic driving, and $\zeta(t)$ is a fluctuating force with zero mean, $\langle\zeta(t)\rangle = 0$, and delta correlated, $\langle\zeta(t)\zeta(t')\rangle = 2D\delta(t-t')$.

The reference, or unperturbed, spatially periodic potential is given by the saw-tooth shape

$$U_0(x) = \begin{cases} \dfrac{x}{\ell}, & 0 \le x < \ell, \\ \dfrac{\lambda - x}{\lambda - \ell}, & \ell < x < \lambda, \end{cases} \tag{2}$$

where λ is the spatial period, and ℓ controls the asymmetry of the potential, which is symmetric for $\ell = \lambda/2$. We will set $\lambda = 1$.

The periodic tilting protocol is

$$F(t) = \begin{cases} \dfrac{A}{\alpha}, & 0 \le t < \dfrac{\alpha}{1+\alpha}\tau, \\ -A, & \dfrac{\alpha}{1+\alpha}\tau < t \le \tau, \end{cases} \tag{3}$$

where τ is the time period, and the parameter α regulates the time symmetry of the force, which is symmetric when $\alpha = 1$.

These simple forms, illustrated in Fig. 1, were chosen because they allow easy symmetry control. Notice that the forces $-U_0'(x)$ and $F(t)$ are unbiased on average, over one spatial and time period, respectively.

We will consider both regular and irregular perturbations $u(x)$ of the reference potential U_0. We will start by analyzing a regular sinusoidal perturbation, such that the total potential, depicted in Fig. 1, is

$$U(x) = [U_0(x) - \varepsilon\cos(2\pi Kx)/2]/N, \tag{4}$$

where $\varepsilon \ll 1$ controls the amplitude of the perturbation and K is an odd integer that controls its wavenumber. We chose K such that extremes of the perturbation coincide with extremes of the potential. Moreover, N is a normalization factor. In Section 4, we will set $N = 1 + \varepsilon$. With this choice, the largest barrier height $\Delta U = U(\ell) - U(\lambda)$ is kept unchanged. This implies a deformation of the potential U_0 besides the addition of roughness. For comparison, in Appendix A, we will show the corresponding main results setting $N = 1$, which means a purely additive perturbation of U_0.

In order to specify the parameter space that will be investigated, the values of ℓ and α, that control the spatiotemporal symmetries, will be kept fixed for three different scenarios that produce net current in the unperturbed case (spatial asymmetry, temporal asymmetry, or both). The spatial period is fixed ($\lambda = 1$). The parameter τ, that controls the time periodicity, will be varied only in the non-adiabatic case addressed in Section 5. Therefore, the parameter space has actually dimension four. Two of the parameters (A and D) are associated to the unperturbed ratchet (tilting and noise amplitudes), while the other two (ε and K) are associated to the rugosity (its amplitude and wavelength).

Other kinds of perturbations (Weierstrass and two-scale functions) will be also considered, as described in Section 4.4.