Physica A 494 (2018) 118–128

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Complexity analysis based on generalized deviation for financial markets

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HIGHLIGHTS

- Investigate the hidden correlation structures.
- Study the return-volatility correlation behaviors.
- Focus on the volatility comparison.

ARTICLE INFO

Article history:
Received 13 August 2017
Received in revised form 5 November 2017
Available online 9 December 2017

Keywords:
Complexity analysis
Generalized deviation
Volatility
Past price
Financial time series

ABSTRACT

In this paper, a new modified method is proposed as a measure to investigate the correlation between past price and future volatility for financial time series, known as the complexity analysis based on generalized deviation. In comparison with the former retarded volatility model, the new approach is both simple and computationally efficient. The method based on the generalized deviation function presents us an exhaustive way showing the quantization of the financial market rules. Robustness of this method is verified by numerical experiments with both artificial and financial time series. Results show that the generalized deviation complexity analysis method not only identifies the volatility of financial time series, but provides a comprehensive way distinguishing the different characteristics between stock indices and individual stocks. Exponential functions can be used to successfully fit the volatility curves and quantify the changes of complexity for stock market data. Then we study the influence for negative domain of deviation coefficient and differences during the volatile periods and calm periods. after the data analysis of the experimental model, we found that the generalized deviation model has definite advantages in exploring the relationship between the historical returns and future volatility.

1. Introduction

Time series analysis is an effective way to describe the operation principle of complex dynamical systems with the popularity of big data. For one thing, time series reflect the output at a high level of complex systems [1,2], such as the circulatory system and ecosystem, for another thing the focus is time series itself, such as financial time series [3–18].

The fundamental concept deviation analysis was proposed by J-P Bouchaud et al. [19], to measure the correlation between future volatility and historical prices. Even earlier, Black studied the financial markets "leverage effect" [20–22], otherwise known as asymmetric volatility, finding that the future price volatility is rising when stock price is falling. The effect is also

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https://doi.org/10.1016/j.physa.2017.12.035
0378-4371/

important for option markets [22–24]. After that, although there are a variety of discussions using Garch-like [25] models to quantify the univariate correlation coefficient of leverage effect [26,27], relevance of the whole space–time structure does not have a quantitative survey. The "leverage effect" economy interpretation is also controversial: Is high volatility of stock leading to price dropping or falling price leading to high volatility of stock [28]?

In 2001, J-P Bouchaud found that the correlations between past price changes and future volatilities can be interpreted by a simple regarded model [19]. The model explained that the absolute magnitude of future price changes (volatility) does not vary with historical price instant change, but with the average historical price changes. Then they proposed to calculate deviation function to quantify magnitude of volatility and discussed the "volatility feedback" mechanism to investigate the relationship between price changes and fluctuations [28–30].

Since the retarded model and volatility feedback mechanism have been applied to various fields [19], it's imperative to promote the accuracy and comprehension of deviation complexity function analysis. In this paper, we propose an improvement method based on deviation complexity function analysis to make the measuring results more general, called generalized deviation complexity analysis. The modified model not only expands the domain of deviation degree successfully but has good quantitative results about the volatility properties in different time periods.

The remainder of this paper is organized as follows. In Section 2, we explain the concept of generalized deviation complexity. Then we propose the generalized deviation complexity analysis method. In Section 3, we test the effectiveness of generalized deviation complexity analysis method with two types of artificial time series: The linear AR model and the ARFIMA stochastic process. In Section 4, application in financial time series is presented, then we analyze the results and make comparison between individual stocks and stock indices. Furthermore, we expand the domain of the deviation coefficient and divide time series into several periods to find more rules of the time series. In Section 5, we give a summary of the model.

2. Generalized deviation complexity of time series

In this section, we use the generalized deviation complexity model to report correlations between past price changes and future volatility both for individual stocks and stock indices. We can consider that the future price volatility is caused not by instantaneous price at some points in past but average level of past prices. Before construction of the model, we make two reasonable predictions: (i) the correlation between past price and future volatility is negative with a positive deviation complexity coefficient because the substantial dropping price often increase the volatility of stock, (ii) the mean reversion mechanism can be used to interpreting price-volatility which means the price will tend to be stable during a long time period [19,28]. We believe that those two predictions are reasonable and efficient because the substantial drop will initiate the market panic which will increase the market volatility. The second prediction is also effective because the stock value will tend to fluctuate around its intrinsic value under long time period. Here, we give the definition of generalized deviation complexity function.

We will call $S_i(t,q)$ the price of stock i at time t with deviation complexity coefficient of q, and $\delta S_i(t,q)$ the (absolute) daily price change. Then we can denote the relative price change:

$$\delta S_i(t,q) = S_i(t+1,q) - S_i(t,q) \tag{1}$$

or

$$\delta \ln S_i(t,q) = \ln S_i(t+1,q) - \ln S_i(t,q) \tag{2}$$

So the generalized deviation complexity function which quantifies the magnitude of future volatility can be expressed as:

$$L_i(\tau,q) = \frac{1}{Z}\left\langle [\delta x_i(t+\tau)]^q \delta x_i(t)^{q-1} \right\rangle \tag{3}$$

where q is a variable number called deviation complexity coefficient used to determine the degree of deviation complexity function. The function measures the correlation between $(q-1)$th power of price changes at time t and qth power of volatility at time $(t+\tau)$. To make results more clear, it is logical to choose $Z = \langle \delta x_i(t,q)^q \rangle^2$ to be the normalization in the formula. When $q = 2$, the model reduces to be retarded model. It can be expressed as:

$$L_i(\tau,2) = \frac{1}{Z}\left\langle [\delta x_i(t+\tau)]^2 \delta x_i(t) \right\rangle \tag{4}$$

This formula is also known as the skewness function. Because of the noisy raw results, we assume that individual stocks behave similarly and average $L_i(\tau,q)$ to get $L_S(\tau,q)$. and $L_I(\tau,q)$ for stock indices. As can infer from the raw results, both $L_I(\tau,q)$ and $L_S(\tau,q)$ are negative, which means that dropping prices often increase the future volatility, also known as the leverage effect. After that, we notice the decay velocity when time interval τ is closing to 0 is quite fast which reflects that big price changes usually following the "rebound" days. So this relation can be fitted quite well by single exponentials:

$$L_{I,S}(\tau,q) = -A_{I,S}\exp(-\frac{\tau}{T_{I,S}}) \tag{5}$$

where A_I and A_S denote the amplitudes of stock indices and individual stocks, T_I and T_S are the decay times, τ is the parameter which represents the interval between past and future. Next, we will test the model through two kind of artificial time series.