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A discrete random walk on the hypercube

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HIGHLIGHTS

- Deriving an analytical solution of mean first-passage time and its scaling of random walks.
- Random walks on the hypercube with a trap.
- Obtaining a rigorous solution of Kirchhoff index for the hypercube.

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ABSTRACT

In this paper, we study the scaling for mean first-passage time (MFPT) of random walks on the hypercube and obtain a closed-form formula for the MFPT over all node pairs. We also determine the exponent of scaling efficiency characterizing the random walks and compare it with those of the existing networks. Finally we study the random walks on the hypercube with a located trap and provide a solution of the Kirchhoff index of the hypercube.

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1. Introduction

A simple symmetric or unbiased random walk on a graph is a walker which moves from one node to another along the edges of the graph, the probabilities of moving along any one of its edges are same. Simple symmetric random walks on graphs may be traced back to the problem of random walk introduced by Pearson [1]. Presently random walks have received considerable attention [2–5] and uncovered a wide range of distinct applications [6–9]. Due to the complexity and variety of real media, it plays an important role in connecting statistical properties of graphs to random walks. A lot of effort has been devoted to the study statistical quantities, such as first-passage time(FPT), global FPT [10], mean FPT(MFPT) [11–13], global MFPT(GMFPT) [14] and so on.

Doyle and Snell [15] have considered underlying electrical network corresponding to a graph by replacing each edge of a graph with a unit resistor. In the underlying electrical network corresponding to a graph, Klein and Randić proposed a useful metric—the resistance distance [16] for the graph. Chandra et al. [17] provided a new approach to compute the MFPT for a connected graph G by giving the following relationship, that is,

$$T_{ij} + T_{ji} = 2E \times r_{ij}, \tag{1}$$

where T_{ij} is the FPT for the walker from nodes i to j, E is the total number of edges of G, and r_{ij} is the effective electrical resistance between nodes i and j of the underlying electrical network corresponding to G. Further, Klein and Randić [16]

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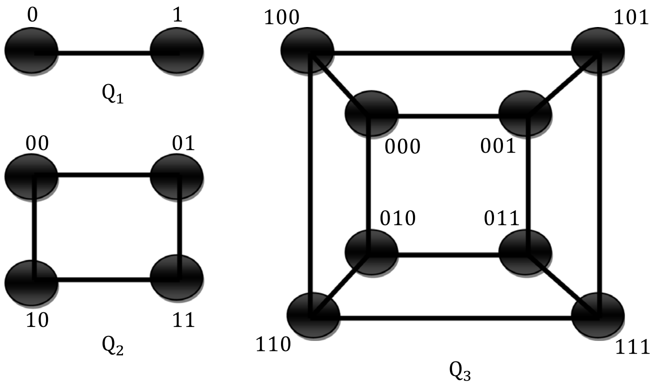

Fig. 1. The hypercubes: Q_1, Q_2 and Q_3.

introduced the sum of resistance distances of all pairs of nodes, i.e.,

$$K_f = \sum_{i<j} r_{ij}, \tag{2}$$

is an important statistical quantity that eventually was named by the Kirchhoff index [18], denoted by $K_f(G)$. For a connected graph with $N \geq 2$ nodes, Zhu et al. [19] and Gutman and Mohar [20] gave another relationship, which is given by,

$$K_f = N \sum_{i=2}^{N} \frac{1}{\lambda_i}, \tag{3}$$

where $\lambda_2, \lambda_3, \ldots, \lambda_N$ are nonzero eigenvalues of the Laplacian matrix $\mathbf{L}$. The element L_{ij} of $\mathbf{L}$ is defined as follows:

$$L_{ij} = \begin{cases} d_i, & \text{if } i = j; \\ -1, & \text{if } i \neq j \text{ and } (i,j) \text{ is an edge in } G; \\ 0, & \text{otherwise,} \end{cases}$$

where d_i is the degree of node i in graph G.

Recently, there has been increasing interest in calculating the MFPT, to obtain the dependence of this primary quantity on the system size. The rigorous results for the MFPT of random walks on regular lattices were obtained by Montroll in his seminal work [21], the exact expressions for the MFPT on fractal networks [22–31] have been obtained. As one of the popular interconnection network model for a parallel computer, the hypercube has been used in both Intel's and NCUBE's computers [32]. Its popularity is due to its good topological properties [33,34]. The random walk on the hypercube approaches the uniform distribution with roughly $\mathcal{O}(N \ln N)$ steps [35] reaching stationarity, where N is the total number of nodes of the hypercube, however few results involve obtaining an exact solution of the MFPT of the hypercube.

In this paper, we derive a closed-form solution for the MFPT over all pairs of nodes of the hypercube, i.e., the GMFPT and obtain the scalings of MFPT. We also obtain the exact expressions for the MFPT with a trap and the Kirchhoff index.

2. Definitions

Let $T_{i,j}$ be the time or steps taken for a walker from nodes i to j. The expected time for a walker to walk from nodes i to j, denoted by $\overline{T}_{i,j} = \mathbb{E}(T_{i,j})$, is called first passage time(FPT), hitting time [23,26,27] or mean first passage time [25]. The average of expected time over all node pairs, denoted by $\langle T \rangle_n = \frac{1}{n(n-1)} \sum_i \sum_{j \neq i} \overline{T}_{i,j}$, is called mean first passage time in [23,26], global mean first passage time (GMFPT). Throughout this paper, we use $\overline{T}_{i,j}$ as the MFPT from nodes i to j, and $\langle T \rangle_n$ as GMFPT.

A g-dimensional hypercube Q_g is defined to be a graph with 2^g nodes in the set $\{0, 1\}^g$. A node x of a hypercube can be encoded by a sequence $x = (x_1, x_2, \ldots, x_n)$, where each x_i is either 0 or 1. Two nodes $x = (x_1, x_2, \ldots, x_g)$ and $y = (y_1, y_2, \ldots, y_g)$ are connected by an edge if and only if x and y differ exactly in one coordinate (see Fig. 1).

Let N_g and E_g be the total number of nodes and edges, it gives

$$N_g = 2^g \tag{4}$$

and

$$E_g = 2^{g-1} g.$$

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