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Nonlinear filtering properties of detrended fluctuation analysis

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HIGHLIGHTS

- Systematic investigation on DFA detrending procedure.
- Statistical properties of sliding window DFA are also investigated.
- A quantitative parameter is proposed to quantify local slope instability.

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ABSTRACT

Detrended fluctuation analysis (DFA) has been widely used for quantifying long-range correlation and fractal scaling behavior. In DFA, to avoid spurious detection of scaling behavior caused by a nonstationary trend embedded in the analyzed time series, a detrending procedure using piecewise least-squares fitting has been applied. However, it has been pointed out that the nonlinear filtering properties involved with detrending may induce instabilities in the scaling exponent estimation. To understand this issue, we investigate the adverse effects of the DFA detrending procedure on the statistical estimation. We show that the detrending procedure using piecewise least-squares fitting results in the nonuniformly weighted estimation of the root-mean-square deviation and that this property could induce an increase in the estimation error. In addition, for comparison purposes, we investigate the performance of a centered detrending moving average analysis with a linear detrending filter and sliding window DFA and show that these methods have better performance than the standard DFA.

1. Introduction

Detrended fluctuation analysis (DFA) has been widely used to assess the presence of long-range correlation and fractal scaling behavior [1–3]. In DFA, to avoid spurious detection of scaling behavior caused by a nonstationary trend embedded in the analyzed time series, a detrending procedure using piecewise least-squares fitting has been applied. An important advantage of DFA over other conventional methods, such as power spectral analysis [4] and rescaled range (R/S) analysis [5], is that it can systematically eliminate nonstationary trends by changing the order of polynomial fitting. In addition, several variants of DFA using different types of detrending techniques have been proposed [6–12]. The performance of DFA and its variants has been tested by a number of numerical studies [3,13–19]. The analytical results of the DFA, such as its asymptotic behavior [20], its relationship with the power spectral density [21], and its relationship with the autocorrelation function [22] have also been reported. However, to lead to better performance of the scaling exponent estimation, criteria

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in selecting a method and a guiding principle to improve the DFA methodology have not been clearly established. Empirical evaluations of the estimation performance have demonstrated that the original DFA remains the method of choice in the scaling analysis [16,17,19].

It has been pointed out that the nonlinear filtering properties of the detrending procedure in DFA may induce instabilities in the scaling exponent estimation [10]. However, an explicit link between the nonlinear filtering and the estimation instabilities has not been clearly elucidated. To understand this issue, we here investigate the adverse effects of the DFA detrending procedure on the statistical estimation, and we demonstrate that the nonlinear filtering properties of DFA results in a nonuniformly weighted estimation of the root-mean-square deviation. In addition, it is shown that this property could induce an increase in the estimation error. From the statistical viewpoint, this property is unfavorable and must be improved.

This paper is organized as follows. In Section 2, we briefly review the scaling analysis method used in DFA. In Section 3, we investigate analytically and numerically the nonlinear filtering properties of DFA. In Section 4, we show numerically that the nonlinear detrending filter of DFA induces an increase in the estimation error. Finally, in Section 5, we conclude by discussing a possible improvement of the DFA methodology.

2. Detrended fluctuation analysis

In this section, we briefly review the standard DFA procedure [1–3]. The scaling analysis method using DFA is the following: 1. The analyzed time series $\{x(i)\}$ of length N is integrated after subtracting the mean from each data point:

$$y(k) = \sum_{i=1}^{k} \left(x(i) - \bar{x}\right), \tag{1}$$

where $\bar{x}$ denotes the sample mean of $\{x(i)\}$. 2. The integrated time series $\{y(k)\}_{k=1}^{N}$ is divided into equal-sized, nonoverlapping segments of length n. 3. In each segment, a polynomial function is fitted to $\{y(k)\}$ by the least-squares method, and then the mean-square deviation from the polynomial fit is calculated. 4. The mean-square deviations are averaged over all segments and its square root $F(n)$, referred to as the fluctuation function, is calculated as

$$F(n) = \left[\frac{1}{N_n}\sum_{i=1}^{N_n}\left\{y(j) - \tilde{y}(j)\right\}^2\right]^{1/2}, \tag{2}$$

where N_n represents the largest possible length and $\tilde{y}(j)$ is the piecewise least-squares-fitting polynomial. Steps 2 and 3 are repeated over multiple scales (window sizes) to explore the relationship between $F(n)$ and n. The slope of a linear relationship between $\log F(n)$ and $\log n$ provides an estimation of the DFA scaling exponent α.

In DFA, the above step 3 is important to remove the adverse effect caused by any nonstationary trend embedded in the analyzed time series. However, as will be analyzed in the next section, this process acts as a nonlinear high-pass filter. Because of this nonlinearity, well-established linear analysis tools, such as the frequency response based on frequency domain analysis, cannot be employed to investigate the methodological properties of DFA. To overcome this difficulty, we recently proposed a method of analyzing the single-frequency response in the time domain [21]. In this paper, using a similar approach, nonlinear filtering properties of DFA will be investigated.

3. Nonlinear properties of the detrending filter in DFA

In this section, we study in detail properties of the detrending procedure in DFA. In addition, for a comparison purpose, we also consider centered detrending moving average (DMA) analysis [10]. By numerical and analytical studies, it has been demonstrated that the performance of DMA is comparable with that of DFA [19]; in particular, mth-order centered DMA, where m is a nonnegative even integer, is very well comparable with that of $(m+1)$th-order DFA [23]. These two methods differ only in the detrending procedure. As shown in Fig. 1(a) and (b), the detrending procedure in DFA acts as a nonlinear high-pass filter. Namely, a single-frequency component [Fig. 1(a)] is deformed into a distorted shape [Fig. 1(b)] through this type filter. Moreover, it generates higher frequency components. As shown in Fig. 2, the power spectrum of the filtered output [Fig. 2(b)] indicates higher frequency components. In contrast, in centered DMA in which a moving average,

$$\tilde{y}(i) = \sum_{j=i-(n-1)/2}^{i+(n-1)/2} y(j), \tag{3}$$

is employed instead of the piecewise least-squares fitting, the detrending procedure acts as a linear high-pass filter [Fig. 1(c) and (d)]. As is already well known [10], the detrending moving average filter has no phase shift, and its frequency response (gain) is given by $1 - \sin(\pi n f)/[n \sin(\pi f)]$.

Fig. 3 provides an example of the estimated trend $\{\tilde{y}(i)\}$ (red dashed lines) when a sample path of Brownian motion (integration of white Gaussian noise) is analyzed by first-order DFA and by centered DMA. As shown in Fig. 3(a), the estimated trend $\{\tilde{y}(i)\}$ (red dashed lines) in DFA shows discontinuous jumps at the end points of each window. Moreover, as