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Statistical analysis of digital images of periodic fibrous structures using generalized Hurst exponent distributions

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HIGHLIGHTS

- Random walk experiments were performed on monochromatic maps.
- Distribution of Hurst exponents represents a given picture.
- The Hurst exponents were calculated using the generalized scheme.
- The generalized Hurst exponent became the useful tool in similar pictures recognition.

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ABSTRACT

Distinction of diverse two-dimensional periodic structures can be based on a large number of methods and parameters, while the quantitative description of differences between similar samples is usually difficult. This article aims, by the use of statistical random walk in a generalized q-order dimensional space, at introducing a methodology to qualify the networked structures on the basis of exemplary textile samples. The presented results were obtained at 1-bit monochromatic maps obtained from optical microscopic pictures. Significant features of samples were represented by the obtained distributions of Hurst exponents and Shannon entropy calculations.

1. Introduction

The concept of diffusion in disordered media attracted in recent years considerable attention, especially in analysis of anomalous transport and diffusion phenomena. It is commonly known that in a homogeneous Euclidean space, using the random walk approach, the observed mean-squared displacement is proportional to the first power of time. In disordered systems, however, including fractals and percolation clusters, this property is no longer valid [1] and the power index equals $2/d_w$ against like in the above standard situation. The $h = 1/d_w$ quantity is called Hurst exponent and d_w is the diffusive dimension. The $h < 0.5$ regime is valid for negative correlations, the $h > 0.5$ case works for positive correlations, during anomalous diffusion process. The $h = 0.5$ case covers the standard random walk and the simple proportionality to time [1–4].

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In addition to random walk experiments the statistical analysis of structural samples can be carried out by many different methods, for example, can be based on diffusion with hopping [5–9], deterministic chaos [10–12], fractal dimension analysis [13] or lacunarity analysis of spatial patters [14,15] to mention some of them.

The presented research efforts are motivated by the introduction of the new concept based on universal fractal properties. We define the algorithms using fractal features of objects, pictures, and images to investigate them and finally classify them. In our research, fractal features of textiles images are investigated using anomalous random walk. The investigation leads to the Hurst exponent and other multifractal hierarchy exponents. This features extraction scheme, for a wide classification of textile images, can be conducted in many research disciplines. This classification may find its own place between other well-established schemes of the machine-learning algorithms [16] such as: SVM (Support Vector Machine), LDA (Linear Discriminant Analysis), PCA (Principal Component Analysis), clustering, Bayesian networks, genetic algorithms, and also in linear and polynomial regression. For example, it can be employed in randomly perturbed systems [17] or to investigate structural stability problems [18,19]. Also, from the fractal processing perspective the presented methodology can find applications in analysis of different physical systems like polymer gels [20–22] or porous media under freezing–thawing conditions [23–25].

The random walk relies on making uncorrelated steps, being a measure of discretized time, with an equal probability, in order to pass a given final distance. This notion can provide clear characteristics of a given physical system even if it requires large number of repetitions and subsequent averages. The method is an example of effective tools in analysis of many transport processes, in inhomogeneous systems and can be generalized into dimensions beyond a classical three-dimensional space. This is what will be presented systematically in the current paper.

Textile fabrics, yarns and fibers, are examples of very interesting, complex networks characterized by many parameters, such as: length, diameter, density of branches, as well as by hairiness [26–30]. The direct, classical microscopic pictures analysis supports practical evaluations, especially estimation of fabrics hairiness [31,32]. Dozens of these different classical methods are applied in textile industry. These methods probe textile surfaces using electromechanical or photomechanical equipment. In haptical tests, for example, a surface is touched and compared with a reference sample, which is always a subjective test. A Confocal Laser Scanning Microscope with depth resolution capability can be used in optical-based roughness tests. Also, two-dimensional CCD images can be utilized for the detection of contamination, breaks, local irregularities, weaving and knitting errors. The purpose of the actual paper is to introduce the most general, statistical and objective approach, which can be used in many different branches of surface technology. We do not want to concentrate on any specific comparative analysis, however we provide some proposals related to the textile materials evaluation.

2. A random walk in general

A basic random-walk statistical experiment, in a one-dimensional space, can be realized by making n unit steps, being a measure of discretized time t, in backward or forward directions, with an equal probability, in order to reach a given final distance x. In this case we have

$$x = \sum_{i=1}^{n} \Delta x_i, \quad \Delta x_i = \pm 1, \tag{1}$$

where Δx_i is the step of a unit length. For a large number of repetitions of the experiment the distance equals zero. However, this is not a case for the average value of the squared distance $\langle x^2 \rangle$, namely

$$\begin{aligned}\langle x^2 \rangle &= \left\langle (\Delta x_1 + \Delta x_2 + \cdots + \Delta x_i + \cdots + \Delta x_n)^2 \right\rangle = \left\langle \left(\sum_{i=1}^{n} \Delta x_i^2 \right) + 2 \left(\sum_{i>j}^{n} \Delta x_i \Delta x_j \right) \right\rangle \\ &= \left\langle n + 2 \left(\sum_{i>j}^{n} \Delta x_i \Delta x_j \right) \right\rangle. \end{aligned} \tag{2}$$

For a pure stochastic phenomenon, when subsequent steps are uncorrelated, the mixed $\Delta x_i \Delta x_j$ term vanishes

$$2 \left(\sum_{i>j}^{n} \Delta x_i \Delta x_j \right) = 0, \tag{3}$$

and we obtain the following simple relation

$$\langle x^2 \rangle = \langle n \rangle = \langle t \rangle . \tag{4}$$

Following these simple introductory derivations, it has to be stated that it is difficult to keep the linear dependence of Eq. (4). If to implement random walk on a non-continuous object, especially on a fractal. In this case, however, Eq. (4) can be modified into the following expression for the averaged squared distance $\langle R^2 \rangle$

$$\langle R^2 \rangle = \langle n \rangle^{2h} \tag{5}$$