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The ultimatum game: Discrete vs. continuous offers

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HIGHLIGHTS

- The ultimatum game is used to study altruistic punishment and rational behavior.
- We model how two groups with different perceptions of fairness might converge.
- We highlight a key distinction between using discrete and continuous monetary offers.
- Modeling discrete offers leads to exponential convergence between groups.
- Modeling continuous offers leads to power-law convergence between groups.

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ABSTRACT

In many experimental setups in social-sciences, psychology and economy the subjects are requested to accept or dispense monetary compensation which is usually given in discrete units. Using computer and mathematical modeling we show that in the framework of studying the dynamics of acceptance of proposals in the ultimatum game, the long time dynamics of acceptance of offers in the game are completely different for discrete vs. continuous offers. For discrete values the dynamics follow an exponential behavior. However, for continuous offers the dynamics are described by a power-law. This is shown using an agent based computer simulation as well as by utilizing an analytical solution of a mean-field equation describing the model. These findings have implications to the design and interpretation of socio-economical experiments beyond the ultimatum game.

1. Introduction

It is quite common for subjects taking part in social science experiments to give or receive monetary compensation. This compensation is offered in discrete units. The reasons for using discrete units stems from convenience and sometimes have deeper psychological underpinnings. However, when one models the subjects' behavior it is often helpful to use continuous variables which are convenient for mathematical treatment, for example for describing dynamics by differential equations. Can the difference between discrete and continuous monetary compensation lead to qualitative differences in the behavior of models describing the experiments? In this paper we shall investigate this question by studying the behavior of an agent based numerical model describing a particular experimental setup (known as the ultimatum game) and compare it to analytical mean-field solutions which treat the monetary compensation in the model either as continuous or as discrete.

The ultimatum game is a widely used experimental setup [1,2], used throughout social sciences, neuroscience and economics to study issues such as rational economic behavior [3], fairness [4–8] and altruistic punishment [9–12]. The

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essential features of the game are deceptively simple. Two players take part in an iteration. The first, called the "proposer", receives a fixed monetary sum (usually \$10 or \$20). She can decide how much of it to keep for herself and how much to propose to the second player (say, keep \$7 to herself and give \$3 to the second player). The second player, known as the "responder", may either accept or reject the proposal. If the proposal is accepted both players walk away with the sums offered by the proposer. On the other hand, if the proposal is rejected by the responder, both receive nothing. Sometimes the proposer or responder is played by the experimenter, trying to check a particular scenario.

If the game is played only once between each pair of players, a rational responder motivated only by monetary considerations should accept any non-zero proposal. Nevertheless, as tens of thousands of subjects all over the world have demonstrated repeatedly, people seem to have an acceptance threshold below which they refuse to accept a proposal, although they suffer monetary loss. Many explanations are proposed for this behavior. Here we are interested in explanations which connect the behavior of the responder with her reputation in a group, and in the behavior of other group members in a similar situation. Indeed, there has been recently experimental effort to understand the influence of exposing a player to information on the behavior of the other participants [13]. Toward this end we formulate a model which assumes that each individual has an internal acceptance threshold. This threshold is influenced by information on previous players' behavior. Thus, if a previous agent accepted a proposal which is below the current player's acceptance threshold, or rejected a proposal above her acceptance threshold then the responder will adjust her acceptance threshold. For lack of concrete experimental evidence on how the acceptance threshold is updated, we assume that it is updated proportionally to the discrepancy.

Such a model naturally makes predictions regarding how two groups with different perceptions of fairness (i.e., acceptance thresholds) will converge to the same acceptance threshold, once they are brought in touch with each other. Our intentions were to investigate this empirically in an experimental setup. Nevertheless, while investigating the numerical behavior of the above described model we noticed a puzzling feature of the model: the acceptance threshold of the two groups converge much more slowly for continuously distributed offers than for a discrete set of possible offers. We show that while the continuous case is characterized by a power-law behavior at long times, the discrete case shows exponential decay at long times. The difference between the two cases becomes clear once the discrete nature of the proposals is resolved, i.e., when the difference between the acceptance thresholds of the two groups is of order of the discreteness of the offers.

In a sense this reminds us of the situation for the estimation of the number of ancestral lineages at past generations given a present-day sample of size, [14] where the functional behavior switches from power-law [15] when the number of generations is large to exponential behavior when the number of generations is of order of one and their discrete nature can no longer be ignored. Similar qualitative changes in the behavior of a system once its discrete nature is explicitly considered have been also reported for molecular reaction diffusion systems [16–18].

This result is interesting by itself, has implications to experimental design in social sciences and economics, and joins the growing recent literature on agent-based simulations of socio-economical behaviors [19]. The paper is organized as follows. In Section 2 we present the details of the agent based model describing the threshold acceptance dynamics of two groups of interacting individuals. A mean-field solution for discrete and continuous offers is derived in Section 3, and compared to the results of the numerical model in Section 4. Situations beyond the mean-field are discussed in Section 5. The last section (Section 6) is devoted to our conclusions.

2. Model

The model is an agent based model [20] in which the threshold acceptance dynamics of two groups of interacting individuals is modeled. The two group populations consist of N individuals (i.e., a total of $2N$ individuals). An individual i belongs to the first group if $1 \leq i \leq N$ or to the second group if $N+1 \leq i \leq N$. Each individual is characterized by an acceptance threshold $g_i(t_k)$ which can take continuous values between 0 and 10, which may change at each time step t_k (where $k = 1 \ldots M$ is the time step, $t = k\Delta t$ and Δt is an arbitrary time interval). Initially, the first group acceptance threshold is distributed around a value $\langle g^{(1)}(0)\rangle$ with a box distribution of width W_1. Similarly, for the second group the average initial value $\langle g^{(2)}(0)\rangle$ with a width W_2.

The dynamics of the model proceed as follows: at initialization, for each agent i an acceptance threshold $g_i(0)$ is drawn from the appropriate distribution. Its value is not changed, unless the agent is chosen at some future time. At each time step t_k an agent i is chosen at random among the N individuals belonging to the same group as the individual in the previous time step t_{k-1} with probability $1-p$, or from the opposite group with probability p. Thus, if $p = 1/2$ there is the same probability for choosing an individual from either group, while for $p < 1/2$ the next individual is from the same group with higher probability. This agent receives information on the previous round at time t_{k-1}, which includes the proposal of the proposer (as described in the previous section, the proposal consists of an offer of the proposer to share a payment of \$10, where \$10-o_{k-1} goes to the proposer and o_{k-1} to the acceptor) and whether it was accepted. The individual i adjusts her acceptance threshold accordingly. If the previous agent accepted a proposal which is below the current agent's acceptance threshold or rejected a proposal above her acceptance threshold then i will adjust her acceptance threshold according to

$$g_i(t_k) = g_i(t_{k-1}) + \xi_k(o_{k-1} - g_i(t_{k-1})), \tag{1}$$

where ξ_k is a random number drawn from a box distribution in the range $[0, X]$. On the other hand if the previous agent accepted a proposal which is above the current agent's acceptance threshold or rejected a proposal below her acceptance threshold then i remains with her current acceptance threshold, i.e., $g_i(t_k) = g_i(t_{k-1})$.