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Bounds for Jeffreys-Tsallis and Jensen-Shannon-Tsallis divergences

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Abstract

Recently has been introduced Jeffreys-Tsallis and Jensen-Shannon-Tsallis divergences, for which we establish new inequalities. Our results refine and generalize recent results in Tsallis theory and one respond to an interesting open problem dated since 2011.

Keywords: Tsallis relative entropy, Jeffreys-Tsallis divergence, Jensen-Shannon-Tsallis divergence, refinement, generalization

1. Introduction

In 1988, C. Tsallis [1] introduced the most interesting extended concept of Shannon entropy by

$$H_q(p) = \sum_{i=1}^{n} p_i \ln_q \frac{1}{p_i}, \quad (q \geq 0, q \neq 1)$$

where $p = (p_1, p_2, ..., p_n)$ is a probability distribution and $\ln_q(x)$ is the q-logarithmic function defined for $x > 0$ as $\ln_q(x) = \frac{x^{1-q}-1}{1-q}$. This function converge to the simple logarithmic function, when $q \to 1$. So the Shannon entropy can be noted as $H_1(p) = -\sum_{i=1}^{n} p_i \log p_i$. In the same manner is defined the Tsallis relative entropy as

$$D_q(p||r) = -\sum_{i=1}^{n} p_i \ln_q \frac{r_i}{p_i},$$

where p, r are two probability distributions. At limit, when $q \to 1$ we have the usual relative entropy(or divergence, or Kullback-Leibler information), $D_1(p||r) = -\sum_{i=1}^{n} p_i \log \frac{p_i}{r_i}$. For $x > 0$, we define here the q-exponential function, the inverse of the q-logarithmic

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