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Intermittent transition between synchronization and desynchronization in multi-regional business cycles

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ABSTRACT

Empirical studies often conclude that multi-regional business cycles exhibit intermittent transition between synchronization and desynchronization of each regional fluctuations. In this study, we robustly observe this behavior (called *chaotic itinerancy*) in a model of multi-regional business cycles, in which all regions of a national economy are homogeneous and connected through producers' behavior based on the average level announced by the government. Although a producer very slowly adjusts his/her output towards the average level, regional business cycles begin to synchronize because of the entrainment effect. Moreover, when a producer emphasizes the profit maximization more and when puts more weight on the average level in his/her decision-making, the economy is more likely to exhibit such intermittent transition. Further, it is clarified that behind intermittent transition exist cycles among periodic orbits with different number of unstable directions.

1. Introduction

Business cycle synchronization has become a topic of growing interest from around the end of the twentieth century (Artis and Zhang, 1999; Imbs, 1999; Selover and Jensen, 1999; Dueker and Wesche, 2003; Süssmuth, 2003, Chap. 5; Baxter and Kouparitsas, 2005; Crowley and Lee, 2005; Savva et al., 2010; Benčík, 2011; Yetman, 2011; Antonakakis, 2012).[2] The vast empirical literature elucidates that countries with intensified trade linkages have resemblant business cycles, which Selover and Jensen (1999) and Süssmuth (2003) explain by proposing a nonlinear mode-locking model. Mode-locking is an inherently nonlinear linkage phenomenon; cycles of different elements are synchronized, that is, attain mode-lock, when the strength of the linkage between oscillating elements reaches a certain threshold.

On the other hand, with the exception of a stylized fact that fluctuations in different regions of a national economy are inclined to synchronize with each other (Rissman, 1999; Carlino and Sill, 2001; Clark and van Wincoop, 2001; Kouparitsas and Nakajima, 2006), little is known about the business cycle synchronization across subnational regions within a country, as mentioned by Kouparitsas and Nakajima (2006). One possible hypothesis about subregional synchronization is that it may occur because of common exogenous shocks such as national fiscal and monetary policies, sudden changes in world commodity prices, trends among consumers, and so on. However, several studies find that common shocks do not seem to be the cause of such synchronization (Carlino and DeFina, 1995; Kozlowski, 1995). A different hypothesis is that the synchronization may be caused by trade linkages between different regions

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2 Synchronization has been paid much attention also in finance after starting the twenty first century. For a survey of related literature, see Huang and Chen (2014).

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of the economy. Another nonlinear mode-locking model is proposed by Selover et al. (2005) under a scenario in which the cycles of different regions synchronize through interregional trade linkages.

In this manner, mode-locking models have been proposed with regard to business cycle synchronization across countries or subnational regions. However, as Süssmuth (2003) properly pointed out, "the idea of 'mode-locking' ... to explain the synchronization of national business cycles ... is not as "new" as Selover and Jensen might have had in mind at the time of their contribution" (p. 70). In fact, we can cite instances of preceding studies, such as Mosekilde et al. (1992,1993), Larsen et al. (1993), Sterman and Mosekilde (1994), Haxholdt et al. (1995), Krugman (1996), and Brenner et al. (1998) although different appellations such as mode-locking, phase-locking, and entrainment are used..

Recent empirical studies often conclude that the regional business cycle exhibits intermittent transition between synchronization and desynchronization of each regional fluctuations (Crowley and Lee, 2005; Savva et al., 2010; Aguiar-Conraria and Soares, 2011; Benčík, 2011; Yetman, 2011; Hanus and Vacha, 2016). The trade linkage hypothesis seems to fail in explaining this fact because trades among countries or subnational regions are supposed to depend mainly upon continuous demand. The main focus of the present paper is to explain the fact by the aid of the model presented by Onozaki et al. (2007). Let us outline their model in the rest of this section.

Since the discovery of synchronization of coupled pendulum clocks by Huygens in the seventeenth century, it has been well-known that multiple oscillators may synchronize if they directly interact with each other. In this sense, a scenario of regional business cycle synchronization through interregional trade linkages is probable and realistic but rather straightforward and obvious. In contrast, another mechanism of regional business cycle synchronization than the direct interaction lies in an economic system: if the government announces the average price and production for all regions at each period and if each regional production is determined based on the preceding information announced by the government, each regional production intertemporally affects all regional productions through information on the average. Thus, there may exist the global, or the all-to-all interaction in an economic system. This mechanism is modeled by Onozaki et al. (2007) by utilizing a system of *globally coupled maps* (GCM). They use nonlinear maps which may behave chaotically depending on parameter values, and illustrate how synchronization occurs and how complex its process is.

It would be better to give a short account of the GCM model here, which is first proposed by Kaneko (1990).[3] It is represented as follows:

$$x_i(t+1) = (1-\varepsilon)f(x_i(t)) + \frac{\varepsilon}{N}\sum_{j=1}^{N} f(x_j(t)),\ i = 1, \ldots, N, \tag{1}$$

where $x_i(t)$ denotes the value of the ith element at discrete time period t, and N the number of elements. A map $f(x)$ describes each element's endogenous dynamics. Usually a noninvertible map is utilized as $f(x)$ that may exhibit chaotic behavior. The second term on the right-hand side of (1) represents the global interaction of each element through the *mean field*, that is, a uniform, all-to-all interaction. Therefore, two opposite effects coexist: the all-to-all interaction is inclined to synchronize all elements and the chaotic instability in each element tends to desynchronize them. Depending on the value of the coupling parameter $\varepsilon \in (0, 1)$, that is, the balance between these two effects, the GCM model exhibits a rich variety of complex phenomena including *chaotic itinerancy* (Kaneko, 1990),[4] which we can regard as the cause of intermittent transition between the synchronization and desynchronization mentioned above. The term chaotic itinerancy describes the phenomenon of an orbit successively itinerating among many ordered states through chaotic transitions in dynamical systems. The phenomenon was independently discovered in a model of optical turbulence by Ikeda et al. (1989), a globally coupled chaotic system by Kaneko (1990), and nonequilibrium neural networks by Tsuda (1990). The terminology was coined by its discoverers to denote universal dynamics in a class of high-dimensional dynamical systems (Kaneko and Tsuda, 2000). In an economic context, Yasutomi (2003) studies the emergence and collapse of money in a computer simulation model from the viewpoint of chaotic itinerancy.

A supplementary remark should be made on synchronization of chaotic systems. It seems that two separate chaotic systems, even though they are identical, cannot synchronize because of an important property of chaotic systems, that is, the sensitive dependence on initial conditions. However, if a small coupling is introduced, chaotic trajectories of those systems tend to synchronize. Two opposite effects coexist in the same way as in GCM model: the coupling effect leads two elements to synchronize and the chaotic instability in each element leads them to desynchronize. Therefore, a persistent aperiodic switching between synchronized and desynchronized states, called *on-off intermittency*, may occur in coupled two chaotic systems. The phenomenon was found by Fujisaka and Yamada (1983) and Yamada and Fujisaka (1983), and is sometimes considered as a special case of chaotic itinerancy when $N = 2$. Bischi et al. (1998), Bischi and Gardini (2000), and Huang and Chen (2014) study synchronization of chaotic oscillators in two-dimensional economic models, all of which exhibit on-off intermittency.

In this study, we use and reexamine the model of Onozaki et al. (2007) and enhance the worth of the model by robustly observing chaotic itinerancy, which they did not observe, for various constellations of parameters as well as various complex phases of regional business cycle synchronization. The remainder of the paper is organized as follows. Section 2 describes a regional business cycle model. Section 3 discusses chaotic itinerancy occurring in the model. Section 4 characterizes chaotic itinerancy from a mathematical point of view. The last section concludes the paper.

2. Model

Although the model to be analyzed in the present paper was already proposed by Onozaki et al. (2007), we repropose it in this section in a slightly different way for the convenience of the reader.

The economy consists of N regions, each with a separate market that is imperfectly competitive. There are n homogeneous producers in each region, and each producer produces homogeneous goods and delivers them only to the market of its own region. Goods are perishable and cannot be carried over to the next period. Consumers are uniformly distributed over all regions and purchase goods from the market they belong to. We assume that there is no interregional trade. Obviously, this assumption is unrealistic, but our main objective is demonstrating that business cycles in different regions may synchronize through producers' behavior based on information announced by the government even if there is no

[3] Coupled map lattices (CML), a framework similar to the GCM model, is also first proposed by Kaneko (1992). The main difference between GCM and CML is that the former includes the mechanism of global interaction of all elements and the latter does not include the mechanism of global interaction but that of local interaction. For an application of CML system in finance, see Huang and Chen (2015).

[4] It is sometimes discussed from the viewpoint of Milnor attractor (Milnor, 1985; Kaneko, 2002).