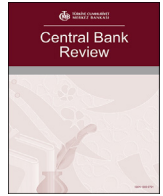Contents lists available at ScienceDirect

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Optimal skill distribution under convex skill costs ☆

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ABSTRACT

This paper studies optimal distribution of skills in an optimal income tax framework with convex skill constraints. The problem is cast as a social planning problem where a redistributive planner chooses how to distribute a given amount of aggregate skills across people. We find that optimal skill distribution is either perfectly equal or perfectly unequal, but an interior level of skill inequality is never optimal.
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1. Introduction

How should income taxes be designed in the face of economic inequality that stems from differences in worker skills? In a seminal analysis, Mirrlees (1971) analyzes this issue and shows that whenever workers' skills are private information, income taxation is distortionary, and optimal income taxation is shaped by a trade-off between equality and efficiency. In this paper, we extend the analysis in Mirrlees (1971) by allowing the government to choose the distribution of skills in the economy in addition to income taxes.

Specifically, we consider a static Mirrleesian economy in which the planner chooses the skill distribution and income taxes. The timing of events is the model is as follows. First, the government chooses a skill distribution taking the average skill level as given. Second, the government chooses the income tax system. Third, agents draw their types from the skill distribution privately. Finally, given their skills, agents work, pay taxes and consume. The main

difference between our model and that of Mirrlees (1971) is the first stage of where, taking the average level of skills as given, the government chooses the dispersion of the skill distribution. Traditional models of Mirrleesian optimal taxation take the distribution of skills in the economy as given, and hence, do not have this initial stage.

We restrict the set of skill distributions available to the government to discrete distributions with two mass points. More precisely, the government chooses mass points w_1, w_2 subject to the following skill constraint,

$$p_1 w_1^\beta + p_2 w_2^\beta = \alpha,$$

where p_1, p_2 are exogenous probabilities attached to the mass points and α is the average skill level in society. The convexity parameter $\beta \geq 1$ controls the technology of skill conversion across agents. When $\beta = 1$, in order to increase type 2 agents' skills by one unit, the government needs to decrease type 1 agents' skills by $\frac{p_2}{p_1}$ units independent of the level of the skill levels. When $\beta > 1$, however, the cost of increasing one type's skills is increasing in that type's skill level. In other words, there is diminishing returns to investing in skills.

In this economy, there are two extremes regarding the dispersion of skills. At one extreme, there is a skill distribution in which $w_1 = w_2$, meaning, all agents have the same earnings capacity. We call this the *perfectly equal* skill distribution. If the government

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chooses this skill distribution, there is no redistributive purpose for income taxation: redistribution is carried out solely via skill distribution choice. At the other extreme, the government can choose a skill distribution in which only one type has positive skills, i.e., $w_1 = 0$ or $w_2 = 0$. Here, a fraction of agents have very high earnings capacity while the rest are completely unproductive. We call this the *perfectly unequal* skill distribution. Redistribution needs to be carried out ex-post in this economy via income taxes. In between the two extreme distributions, there is a continuum of skill distributions with different levels of skill dispersion.

In a closely related paper Leung and Yazici (2017), we analyze the optimal skill distribution problem in a similar framework under the assumption that $\beta = 1$, i.e., the planner faces a linear skill constraint. There, we prove that, whenever $\beta = 1$, the socially optimal skill distribution is always perfectly unequal, i.e., $w_i = 0$, for some i. The main novelty of the current paper over Leung and Yazici (2017) is that here we allow for $\beta > 1$, meaning we allow for convex skill distributions. This is an important generalization. As discussed earlier, when $\beta > 1$, the cost of increasing one type's skills is increasing in that type's skill level. In other words, $\beta > 1$ case is akin to the assumption of diminishing returns to investing in skills, and there is a large body of empirical evidence that supports the notion that human capital investment features diminishing returns.[1]

We show that under full information, the socially optimal skill distribution is either perfectly unequal or perfectly equal, depending on the convexity of the skill constraint and the convexity of the disutility function. When there is private information about skills, we provide a sufficient condition for the optimality of perfectly equal skill distribution that depends on the convexity of the skill constraint and the convexity of the disutility function. When this condition does not hold, it is hard to provide an analytical solution. Instead, we parameterize the utility and disutility functions, and solve the optimal skill distribution problem numerically. We find that the socially optimal skill distribution is again either perfectly equal or perfectly unequal. In this case, we observe that, in addition to the convexity of the skill constraint and the convexity of the disutility function, the level of concavity of the utility function also matters for whether perfectly equal or unequal distribution is optimal.

This paper is also closely related to Cremer et al. (2011). Like Leung and Yazici (2017), Cremer et al. (2011) assume a linear skill constraint and show that the perfectly unequal skill distribution provides strictly higher social welfare than perfectly equal skill distribution. In the current paper, we go beyond the linear skill constraint assumption and provide an analysis of optimal skill distribution under diminishing returns to skill acquisition. Boadway and Pestieau (2006), Simula (2007), and Hamilton and Pestieau (2005) analyze comparative static properties of optimal allocations with respect to certain parameters of the skill distribution.

The rest of this paper is structured as follows. In Section 2, we introduce the model formally. In Section 3, we characterize optimal skill distribution both in the cases in which skills are public and private information. Section 4 provides concluding remarks.

2. Model

There is a unit measure of agents. They produce output individually according to the production function

$$y = wl,$$

where y denotes output, w denotes skill level, and l denotes labor effort.

Each agent's preference is given by

$$u(c) - v(l),$$

where c is consumption and u and v satisfy $u', -u'', v' > 0$ and $v'' > 0$.

The novelty of our analysis is that we allow the government to choose the distribution of skills. For simplicity, it is assumed that skills can take only two values, w_1 and w_2. The probability of drawing w_1 is p_1 and the probability of drawing w_2 is p_2. We allow the government to choose w_1 and w_2 subject to the given total skill level α, but p_1 and p_2 are exogenously given. In other words, the government chooses w_1 and w_2 subject to the following skill constraint:

$$p_1 w_1^\beta + p_2 w_2^\beta \leq \alpha.$$

Allocation. An *allocation* in this economy is defined as $(w_i, c_i, l_i)_{i=1,2}$, where c_i and l_i represent consumption and labor allocation of type i.

Feasibility. An allocation is *feasible* if

$$p_2 c_2 + p_1 c_1 \leq p_2 w_2 l_2 + p_1 w_1 l_1, \tag{1}$$

$$p_1 w_1^\beta + p_2 w_2^\beta \leq \alpha, \tag{2}$$

$$w_1, w_2, c_1, c_2, l_1, l_2 \geq 0. \tag{3}$$

The first inequality means that total consumption cannot exceed total output. The second inequality ensures that the average skill level of the distribution chosen by the government does not exceed α. Finally, the third inequality is just the non-negativity of skill, consumption and labor allocations.

The timing of the events is as follows. First, the government chooses the skill distribution. Then, the government chooses a tax function $T : \mathscr{R}_+ \to \mathscr{R}$, where $T(y)$ is the income tax that an agent with income y pays. Then, each agent privately draws her skill from the chosen skill distribution. Finally, each agent chooses his optimal consumption and labor allocation given the tax system. Taking w_i as given, agent i solves the following consumption-labor problem.

$$\begin{aligned} &\max_{c_i, l_i} u(c_i) - v(l_i) \\ &s.t. \\ &c_i \leq w_i l_i - T(w_i l_i). \end{aligned} \tag{4}$$

Government's Optimal Tax Problem. The government chooses the distribution of skills and the tax function to maximize the total welfare in the economy subject to the fact that the resulting allocation solves each agent's problem.

$$\begin{aligned} &\max_{w_1 \geq 0, w_2 \geq 0, T(\cdot)} p_2[u(c_2) - v(l_2)] + p_1[u(c_1) - v(l_1)] \\ &s.t. (2) \text{ and} \\ &\text{for each } i, (c_i, l_i) \text{ solves } (4). \end{aligned} \tag{5}$$

Social Planning Problem. By the Revelation Principle, the government's optimal tax problem given by (5) is equivalent to a social planning problem in which a planner chooses the skill distribution and the consumption and labor allocations directly as functions of agents' types. The assumption that taxes only depend on income and not on agents' types in the optimal tax problem introduces the restriction that agents' types are private information