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An alternative mean reversion test for interest rates

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A B S T R A C T

A number of empirical studies assert that interest rates are governed by unit root processes rejecting any form of reversion to a long term mean by resorting to certain tests, among which the Augmented Dickey Fuller (ADF) is the most widely used one. In this study, we propose an alternative testing methodology that can be applied along with ADF test, in the sense that there are times where it can capture stationarity when the other fails to do so. Moreover, our test has more power than ADF test. As an application to real-data, we consider 10-year US and Turkish T-bond rates.

1. Introduction

The assertion whether interest rates possess a unit root is a widely explored issue in the literature. Often researchers have come up with the conclusion that interest rates are not stationary. Rose (1988) applied traditional ADF (Dickey and Fuller, 1979) and Philips-Perron (PP) (1988) unit root tests for 18 OECD countries and concluded that the nominal interest rates are not stationary. MacDonald and Murphy (1989) took three-month T-bill rates for Belgium, Canada, United Kingdom and United States between 1955 Q1-1986 Q4 (1957 Q1 to 1986 Q4 for Belgium) and failed to reject the presence of unit root. Siklos and Wohar (1997) considered one, three, six and twelve-month Euro deposit rates for 10 countries and came up with the same result. The conclusions were developed usually through the application of ADF test. Some authors, on the other hand used alternative panel data unit root tests (see for instance Wu and Zhang, 1997; Wu and Chen, 2001) and asserted that short-term interest rates sometimes exhibit mean reversion. End (2011) considered an extremely long period (two hundred years of interest rate data of the Netherlands, Germany, US and Japan) and could not encounter mean reversion. He only could find traces with some smooth transition autoregressive (STAR) framework.

Once the data in question is not stationary, this problem is usually overcomed via differencing the data until stationarity is achieved. However, according to Brooks (2014), differencing a stationary series results in loss of information and observations. In order not to lose any information, it is crucial to detect stationarity, whenever interest rates indeed return to a constant mean. Ours is a humble effort to suggest a mean-reversion test which is applicable to interest rates. The starting point of the work was the observation that in many empirical studies interest rates are differenced, whereas there are good reasons to expect interest rates to revert to a mean, especially in the long run. First of all, nominal interest rates

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can be decomposed into expected inflation and real interest rates. The latter is composed of risk,[1] term[2] and liquidity premia.[3] When sub-components of nominal interest rates are examined we see that a non-stationary change in price level (inflation or deflation) has many negative implications as mentioned in the booklet "Inflation and Price Stability" by the Central Bank of the Republic of Turkey (2014). As a result, central banks take various actions to stabilize the domestic price level, which in general is a pre-determined target equivalently a mean where inflation is desired to return. As to the sub-components of the real interest rate; high risk premium is closely related with debt sustainability issues and at some point, this forces the issuer to take corrective measures such as cutting expenditures or extending debt maturities. High term premium is associated with low investor confidence, and again necessary actions need to be taken to restore confidence in the market. Finally, liquidity premium is a sign of shallow financial markets, and market authorities can modify regulations, or boost domestic savings in order to improve financial depth.

From another point of view, due to no-arbitrage conditions, the term structure of interest rates moves in tandem, so that short term interest rate movements are transmitted to longer maturities. Yet, central banks exert regulations on short term rates in order to control inflation, boost growth, reduce unemployment or achieve any other macroeconomic target. Thus, short rates and longer maturities tend to fluctuate around some long-run mean.

In order to get a mathematical idea about mean reversion, we resort to the most analytically tractable and intuitive stochastic interest rate model incorporating mean reversion, which is Vasicek (1977).[4] Accordingly, the interest rates are governed by Eq. (1):

$$dr_t = \theta(\mu - r_t)dt + \sigma dW_t \tag{1}$$

where μ is the long term mean, θ is the speed of mean reversion, σ is the volatility and W_t is the standard Wiener process. In this model, the interest rates revert to the long term mean μ because the further the interest rate diverges from the mean, the stronger they are pulled down to move towards it according to the magnitude of θ.

Our main contribution is to propose an alternative testing procedure in order to analyze the situations where classical linear tests fail to reject the presence of unit root. We convert the OU process into a linear regression framework, where we can easily compute the *t*-statistics of the variable pertaining to the mean reversion test and compare it with the critical values we have calculated via Monte Carlo simulation based on the work of Szimayer and Maller (2004), which presents the theoretical asymptotic value of the OU test, under the null hypothesis H_0, for the case of no mean reversion.

From the discussions above, it is plausible to examine the connection between mean reversion and unit root tests. Any time series with a unit root becomes non-stationary as the effects of shocks are accumulated and build up a stochastic trend. In case of stationary time series, the effects of past shocks will die out. In that sense, unit root tests may also be regarded as a test of the absence of mean reversion for the underlying time series. However, in practice unit root tests, particularly the ADF test, might fail to reveal the mean-reversion of a time-series, when the data generating process is indeed reverting to mean. When we ran simulations, we observed that both tests make similar number of Type-I errors. However, when the simulated data is generated by an OU process we found out that our test outperforms ADF test in terms of Type-II error which indicates that it has more power.

Rest of the study is as follows: In Section 2 we briefly describe ADF and OU tests together with some derivations and comparisons. Section 3 is devoted to applications. Finally, section 4 concludes.

2. ADF and OU tests

Functional central limit theorem (FCLT) is essential for ADF test statistics. Theorem 1 is by Donsker (1951, 1952):

Theorem 1. Take independent and identically distributed variables ε_t with a zero mean and a variance $\sigma^2 < \infty$. Consider the following partial sum $S_T(r) = \sum_{t=1}^{[rT]} \varepsilon_t$ where $r \in [0,1]$ and $\lfloor . \rfloor$ denotes the integer part. Now the scaled version of the partial sum converges in distribution to Brownian motion that is

$$Z_T(r) = S_T(r)/\sigma\sqrt{T} \overset{d}{\Rightarrow} B(r).$$

Eq. (2) and Eq. (3) are essential for the determination of Dickey-Fuller distribution, where y_t is the data with certain time-series dynamics and ε_t are the shocks:

$$T^{-1}\sum_{t=1}^{T} y_{t-1}\varepsilon_t \overset{d}{\Rightarrow} \int_0^1 B(r)dB(r) = \frac{1}{2}\sigma^2\left[B(1)^2 - 1\right] \tag{2}$$

$$T^{-1}\sum_{t=1}^{T} y_{t-1}^2 \overset{d}{\Rightarrow} \int_0^1 B(r)^2 dr = \sigma^2 \int_0^1 B(r)^2 dr \tag{3}$$

Through consideration of an AR (1) process, after some calculations the ADF test distribution can be computed as:

$$F\left(\hat{\delta}\right) = \frac{\int_0^1 B(r)dB(r)}{\int_0^1 B(r)^2 dr} = \left(\frac{1}{2}\right)\frac{\left[B(1)^2 - 1\right]}{\int_0^1 B(r)^2 dr} \tag{4}$$

For demeaned and de-trended Brownian motions we replace $B(r)$ with

$$B(r)_\mu = B(r) - \int_0^1 B(s)ds \tag{5}$$

$$B(r)_\beta = B(r) - (6r-4)\int_0^1 B(s)ds - (12r-6)\int_0^1 sB(s)ds \tag{6}$$

(for detailed proofs and derivations see Patterson, 2010).

For calculation of the asymptotic distribution of the OU test we follow the methodology proposed by Szimayer and Maller (2004):

The OU process is defined by

$$dX_t = \theta(\mu - X_t)dt + \sigma dB_t \tag{7}$$

and it admits a unique solution via integration by parts as:

[1] Risk premium is an additional required rate of return in order to hold a riskier bond than any other reference bond.

[2] Term premium is an additional required rate of return in order to hold a bond of a longer maturity than any other reference bond.

[3] Liquidity premium is an additional required rate of return in order to hold a less liquid bond compared to any other reference bond.

[4] Vasicek's model is an application of Ornstein and Uhlenbeck (1930) process to interest rates.

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