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Short-term probabilistic forecasting of wind speed using stochastic differential equations

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ABSTRACT

It is widely accepted today that probabilistic forecasts of wind power production constitute valuable information that can allow both wind power producers and power system operators to exploit this form of renewable energy economically, while mitigating the potential adverse effects relating to its variable and uncertain nature. In order to provide reliable wind power forecasts for periods beyond a couple of hours, forecasts of the wind speed are fundamental. In this paper, we propose a modeling framework for wind speed that is based on stochastic differential equations. We show that stochastic differential equations allow us to capture the time dependence structure of wind speed prediction errors naturally (from 1 to 24 h ahead) and, most importantly, to derive point and quantile forecasts, predictive distributions, and time-path trajectories (also referred to as scenarios or ensemble forecasts), all using one single stochastic differential equation model that is characterized by a few parameters.

1. Introduction

The last few years have witnessed a remarkable increase in the contribution of renewable energy sources to the global electricity supply, with the largest share in many countries coming from wind turbines (The European Wind Energy Association, 2013). However, wind power production is highly variable and uncertain, thus challenging traditional practices for power system operation and the trade of this form of renewable energy in electricity markets. In order to mitigate the adverse effects of the stochastic nature of wind, good forecasts of the power generated by wind farms are a must. Furthermore, to manage and exploit wind energy fully, these forecasts should provide power system operators and wind power producers not only with a single-valued guess of the future wind generation – a so-called *point forecast* – but also with information on possible outcomes and their associated probabilities of occurrence. This enriched form of forecasting is known as *probabilistic forecasting*, and takes on its full meaning in the context of wind power management and trading (Morales, Conejo, Madsen, Pinson, & Zugno, 2014; Zhou et al., 2013).

The literature on wind power forecasting is now vast, but is centered mostly on techniques for producing point predictions. For a comprehensive review of the topic, the interested reader is referred to the work of Costa et al. (2008), Monteiro et al. (2009) and Foley, Leahy, Marvuglia, and McKeogh (2012). On the other hand, methods for wind power probabilistic forecasting are not so well developed. Wind power density forecasts are commonly obtained by superimposing a model for the probability distribution of prediction errors on a point forecast (typically, the average or most likely outcome; see Bremnes, 2006; Møller, Nielsen, & Madsen, 2008; Pinson & Kariniotakis, 2010), or by post-processing ensemble forecasts from meteorological models so that they represent the true predictive density (Nielsen et al., 2006; Pinson & Madsen, 2009; Taylor, McSharry, & Buizza, 2009). In the realm of probabilistic

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forecasting, it is customary to distinguish between *parametric* and *nonparametric* methods. The former presuppose a certain standard distribution for the forecast error, and as a result, the modeling endeavor boils down to estimating the parameters that characterize such a distribution (Messner, Zeileis, Broecker, & Mayr, 2013; Pinson, 2012; Thorarinsdottir & Gneiting, 2010; Thorarinsdottir & Johnson, 2012). In contrast,nonparametric methods do not assume any pre-specified forecast error distribution and work directly with the empirical distribution instead (Bessa, Miranda, Botterud, Zhou, & Wang, 2012; Bremnes, 2004; Messner et al., 2013; Pinson, Nielsen, Møller, Madsen, & Kariniotakis, 2007). Recent work has also focused on generating realistic sample trajectories of the stochastic process and producing multi-horizon or multivariate probabilistic forecasts. To this end, the most popular approach is to fit a marginal predictive density for each univariate output variable (e.g., the wind speed in each time period of the prediction horizon) – see Lerch and Thorarinsdottir (2013) for a comparison of different regression-type models for wind speed – and then combine these marginals into a multivariate cumulative density using copula theory (Schefzik, Thorarinsdottir, & Gneiting, 2013).

Forecasts of wind power are improved markedly by the use of numerical weather predictions (NWPs) of wind speed. Foley et al. (2012) provide an overview of the different uses of NWPs for wind power forecasting. Specifically, they underline the fact that ensemble forecasts of wind speed can be used to obtain valuable information on the reliability of the wind power forecast. The usefulness of NWPs of wind speed for wind power forecasting has also been stressed, for instance, by De Giorgi, Ficarella, and Tarantino (2011) and Ramirez-Rosado, Fernandez-Jimenez, Monteiro, Sousa, and Bessa (2009). Taylor et al. (2009) and Pinson and Madsen (2009) use predictive densities of wind speeds to produce probabilistic forecasts of wind power. These predictive densities are estimated from the ensemble forecasts provided by a NWP system.

This paper describes a novel approach to wind speed probabilistic forecasting based on stochastic differential equations (SDEs). The proposed SDE model upgrades numerical weather predictions using wind speed data and provides plausible time-path trajectories of the wind speed process that are perfectly comparable to the ensemble forecasts obtained from a NWP system. Furthermore, our SDE model can generate these trajectories for a specific wind site swiftly. For these reasons, our SDE model can contribute significantly to wind power forecasting. More generally, SDEs offer a powerful and versatile modeling framework that allows us to issue point forecasts and all forms of probabilistic forecasts (namely quantiles, densities, and time-path trajectories) consistently using the same model. Moreover, the parameters characterizing the SDE can be interpreted intuitively, which makes it much easier to formulate model extensions on the basis of the specific physics of the underlying stochastic process or of observable statistical deficiencies. The proposed modeling framework naturally captures the time dependence of forecast errors and events with zero probability, such as negative wind speeds, and does not need to assume Gaussian innovations. Seen in a broader perspective, SDEs cover the large class of stochastic processes with continuous trajectories, and, in fact, many of the discrete time models used in classical time series theory can be seen as discrete-time versions of SDEs.

The application of stochastic differential equations to forecasting, and to wind power forecasting in particular, is a very recent topic, and consequently, the technical literature in this regard is scant. However, two studies should be mentioned here, namely those of Møller, Pinson, and Madsen (2013) and Zárate-Miñano, Anghel, and Milano (2013). SDEs are used fruitfully by Møller et al. (2013) for wind power forecasting, considering state-dependent diffusions and with a numerical weather prediction as external input. The SDE model proposed by the authors is fitted simultaneously to data from 1 to 48 h ahead, which makes it computationally intensive to estimate and limits the amount of data that can be used for fitting. In contrast, the SDE model proposed in this paper is estimated on one-step-ahead data, and thus, the approach that we use here resembles that of Box–Jenkins type models that are fitted using one-step-ahead information. Furthermore, our model focuses on wind speed, not wind power, and therefore the challenges are different. Zárate-Miñano et al. (2013) present a continuous time model for wind speed, which aims to simulate wind speed trajectories over very short time horizons. However, it cannot be used for forecasting, as the parameters are not estimated, no external inputs are allowed, and the SDE model is limited to a very simple structure. In addition, the SDE model that Zárate-Miñano et al. (2013) propose is designed to fit the long-term stationary distribution of wind speed. The stationary distribution, however, does not necessarily hold for the short term, as one can see for the case of the climatological forecast.

The approach to modeling wind speed taken in this paper is similar to the approach to modeling solar irradiance adopted by Iversen, Morales, Møller, and Madsen (2014), in that they both rely on SDEs. However, the weather phenomena considered in these two papers, namely wind speed and solar irradiance, are remarkably different, and each has its own challenges. Indeed, the modeling of wind speed differs substantially from that of solar irradiance in terms of the physical domain of the underlying process and its periodic nature. This results in distinct structures for the drift and diffusion terms. Moreover, in the case of wind speed, the mere introduction of the NWP as an exogenous input to the SDE model is not advantageous, as it results in the simulated process systematically lagging behind the NWP. This will be shown later on. We solve this issue by introducing the derivative of the NWP as well. Furthermore, in the present paper, we provide a straightforward methodology for generating predictive densities and time-path trajectories of wind speeds that is analogous to the ensemble forecasts obtained from a NWP system. These ensemble forecasts are of particular relevance for wind power forecasting, whereas time-path trajectories and predictive densities for solar irradiance are still of limited use, at least comparatively speaking.

The remainder of the paper is organized as follows: Section 2 provides a short introduction to stochastic differential equations (SDEs) and the parameter estimation procedure. Section 3 provides a model for wind speeds based