



Contents lists available at ScienceDirect

International Journal of Forecasting

journal homepage: www.elsevier.com/locate/ijforecast

Macroeconomic forecasting and structural analysis through regularized reduced-rank regression

Emmanuela Bernardini^a, Gianluca Cubadda^{b,*}

^a Banca d'Italia, Roma, Italy

^b Dipartimento di Economia e Finanza, Università di Roma "Tor Vergata", Via Columbia 2, 00133 Roma, Italy

ARTICLE INFO

Keywords:

Canonical correlation analysis
Vector autoregressive models
Shrinkage estimation
Macroeconomic prediction

ABSTRACT

This paper proposes a strategy for detecting and imposing reduced-rank restrictions in medium vector autoregressive models. It is known that Canonical Correlation Analysis (CCA) does not perform well in this framework, because inversions of large covariance matrices are required. We propose a method that combines the richness of reduced-rank regression with the simplicity of naïve univariate forecasting methods. In particular, we suggest the usage of a proper shrinkage estimator of the autocovariance matrices that are involved in the computation of CCA, in order to obtain a method that is asymptotically equivalent to CCA, but numerically more stable in finite samples. Simulations and empirical applications document the merits of the proposed approach for both forecasting and structural analysis.

© 2013 International Institute of Forecasters. Published by Elsevier B.V. All rights reserved.

1. Introduction

In principle, the existence of co-movements among economic time series implies that multivariate forecasting methods should outperform univariate techniques. Indeed, this is the main theoretical motivation for the development of models for large multivariate data sets, such as factor models (Forni, Hallin, Lippi, & Reichlin, 2000, 2005; Stock & Watson, 2002a,b) and Bayesian vector autoregressions (Banbura, Giannone, & Reichlin, 2010). In practice, however, univariate forecasting models are hard to beat, particularly for short horizons. In a recent paper, Carriero, Kapetanios, and Marcellino (2011) explore the merits of the best available technology for forecasting large datasets, and conclude that, for one- and two-step-ahead forecasts, there are no multivariate models that are able to beat the univariate autoregressive benchmark. One possible

explanation of this finding is that the existence of common components among time series leads to simple univariate models (Cubadda, Heccq, & Palm, 2009) which can be identified and estimated easily, whereas the efficient multivariate modelling of many time series is a hard task.

In a vector autoregressive (VAR) framework, the presence of common components is equivalent to imposing proper reduced rank structures on the model coefficient matrices. At the statistical level, these restrictions can be tested for and imposed on the estimation by means of Reduced-Rank Regression (RRR) techniques, see *inter alia* Cubadda (2007) and the references therein.

We propose a method that combines the richness of RRR with the simplest univariate forecasting method, i.e., each series is forecast by its unconditional mean. To this end, we resort to a proper shrinkage estimator of the autocorrelation matrices for computing RRR, in place of the natural estimator. The resulting estimator is asymptotically equivalent to the Maximum Likelihood (ML) solution, but it is numerically more stable in finite samples.

In order to check the practical value of the proposed method, we consider its performances when applied to medium VAR models. Indeed, Banbura et al. (2010) and

* Corresponding author.

E-mail addresses: emmanuela.bernardini@bancaditalia.it (E. Bernardini), gianluca.cubadda@uniroma2.it (G. Cubadda).

URL: <http://www.economia.uniroma2.it/nuovo/facolta/docenti/curriculum/GianlucaCubadda.html> (G. Cubadda).

Koop (2013) provide convincing evidence that no substantial predictive gains are obtained by increasing the dimension of the VAR beyond 20. Hence, we focus on reduced-rank VAR models, where the number of predictors is considerably larger than in typical small-scale forecasting models, but not large enough to resort to statistical inference that is based on methods involving double asymptotics (see for example Cubadda & Guardabascio, 2012). We show, by both simulations and empirical applications, that the proposed approach performs well with respect to traditional medium-size macroeconomic methods, and demonstrate that our new method improves both the forecasting and estimation of structural parameters.

The paper is organized as follows. Section 2 discusses the theoretical aspects. Section 3 compares various forecasting procedures in an empirical application to US economic variables. Section 4 uses simulations to evaluate the merits of the various methods in terms of model specifications, forecasting performances, and precision in estimating structural parameters. Section 5 draws some conclusions, and the proofs of the theorems are reported in the Appendix.

2. Theory

We start this section by briefly reviewing the reduced-rank VAR model for stationary time series (see Reinsel & Velu, 1998, and the references therein) and the associated estimation issues.

Consider an n -vector time series y_t generated by the following stable VAR(p) model:

$$y_t = \sum_{i=1}^p \Phi_i y_{t-i} + \varepsilon_t = \beta' x_{t-1} + \varepsilon_t, \quad t = 1, 2, \dots, T,$$

where β is a $pn \times n$ coefficient matrix, $x_t = [y_t', \dots, y_{t-p+1}']$, and ε_t are i.i.d. innovations with $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_t') = \Sigma_{\varepsilon\varepsilon}$ (positive definite), and finite fourth moments. For the sake of simplicity, no deterministic terms are included.

If we assume that the series y_t exhibits the serial correlation common feature (Engle & Kozicki, 1993; Vahid & Engle, 1993), we can rewrite the VAR as an RRR model

$$y_t = A\psi'x_{t-1} + \varepsilon_t = AF_{t-1} + \varepsilon_t,$$

where A and ψ are, respectively, full-rank $n \times q$ and $np \times q$ matrices, and $F_t = \psi'x_t$ are q common factors.

One well-known method of obtaining the factor weights ψ is Canonical Correlation Analysis (CCA). In particular, the matrix ψ lies in the space generated by $[v_1, \dots, v_q]$, where v_i ($i = 1, 2, \dots, q$) is the eigenvector associated with the i th largest eigenvalue of the matrix

$$\Sigma_{xx}^{-1} \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx},$$

where $\Sigma_{xx} = E(x_{t-1}x_{t-1}')$, $\Sigma_{yy} = E(y_t y_t')$, and $\Sigma_{xy} = E(x_{t-1}y_t')$. Finally, the matrix A is obtained (up to an identification matrix) by regressing y_t on the q canonical factors $[v_1, \dots, v_q]'x_{t-1}$, see, *inter alia*, Anderson (1984) for further details.

As a statistical method, CCA has numerous merits, of which the most important are that it provides the ML

solution under the Gaussianity assumption and is invariant to non-singular linear transformations of both y_t and x_{t-1} . Moreover, Carriero et al. (2011) have recently extended consistency results for the case that the number of predictors $N = np$ diverges more slowly than T .

However, CCA suffers from some limitations when the system dimension is large. First, CCA is unfeasible when the number of observations T is less than the number of predictors N . Second, even when T is large, the CCA solution is numerically unstable and statistical inference is unreliable in a medium N framework, see Cubadda and Hecq (2011).

We propose to solve these problems by shrinking the sample autocorrelation matrix of series y_t , which CCA is based on at the sample level. In particular, CCA is usually performed by solving the eigenvector equation $S_{yy}^{-1}S_{yx} \widehat{v} = \widehat{\lambda} \widehat{v}$, where

$$S = \begin{bmatrix} S_{yy} & S_{yx} \\ S_{xy} & S_{xx} \end{bmatrix}$$

is the sample covariance matrix of the series $w_t = [y_t', x_{t-1}']$. We suggest that a proper shrinkage estimator of the covariance matrix $\Sigma = E(w_t w_t')$ be used in place of S .

In general, a shrinkage estimator (Stein, 1956) is an optimally weighted average of two existing estimators, an asymptotically unbiased estimator that suffers from a large estimation error, and a target one that is biased, but with a lower estimation error. We propose a regularized version of CCA that requires the solution of the eigenvector equation $Z_{xx}^{-1}Z_{xy}Z_{yy}^{-1}Z_{yx}\tilde{v} = \tilde{\lambda}\tilde{v}$, where

$$Z = \alpha D + (1 - \alpha)S,$$

D is a diagonal matrix having the same diagonal as S , and $\alpha \in [0, 1]$.

Note that when $\alpha = 1$, the full-rank regression case coincides with n univariate white noises, whereas when $\alpha = 0$, we go back to the usual CCA solution. Hence, in a way, we are shrinking RRR towards the simplest forecasting univariate model, i.e., the white noise. When variables are made stationary by differencing, our target is equivalent to the so-called Minnesota prior in Bayesian VAR modelling, where the forecasts of the time series levels are shrunk towards random walks (see, *inter alia*, Litterman, 1986).

In the choice of the optimal α , we follow the data-based approach of Ledoit and Wolf (2003), which has the advantage of providing a closed form solution to the optimization problem. In particular, Ledoit and Wolf (2003) propose the minimization of a risk function based on the Frobenius norm of the difference between the shrinkage estimator Z and the covariance matrix Σ , i.e.,

$$\begin{aligned} R(\alpha) &= E(\|Z - \Sigma\|^2) \\ &= E \left[\sum_i^{N+n} \sum_j^{N+n} (\alpha d_{ij} + (1 - \alpha)s_{ij} - \sigma_{ij})^2 \right], \end{aligned}$$

and the solution to this optimization problem is

$$\alpha^* = \frac{\sum_{i \neq j} \text{Var}(s_{ij})}{\sum_{i \neq j} [\text{Var}(s_{ij}) + \sigma_{ij}^2]},$$

Download English Version:

<https://daneshyari.com/en/article/7408362>

Download Persian Version:

<https://daneshyari.com/article/7408362>

[Daneshyari.com](https://daneshyari.com)